Quiz 1
Thursday, October 8th
7:30 PM – 9 PM

Please, answer the following questions. Write your answers directly on the quiz. You can achieve a total of 100 points. There are 5 multiple-choice questions, followed by 2 free response questions. You should read all of the questions first.

Good luck!

NAME: Scott Young

MIT ID NUMBER:

TA:

CLASS TIME:

EMAIL:

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1 Multiple Choice (30 points)

Answer the following questions. You DO NOT need to justify your answer.

1. (6 Points) Consider an economy with two goods and two periods. Data are

   Good 1
   
   \[ p^1_t = 1 \]
   
   \[ p^2_{t+1} = 1 \]
   
   \[ q^1_t = 1 \]
   
   \[ q^1_{t+1} = 1.1 \]
   
   Good 2
   
   \[ p^2_t = 1 \]
   
   \[ p^2_{t+1} = 1.4 \]
   
   \[ q^2_t = 1 \]
   
   \[ q^2_{t+1} = 1.3 \]

   where \( q^1_t \) stands for the quantity of good 1 produced in period \( t \), \( q^2_t \) stands for the quantity of good 2 produced in period \( t \), \( p^1_t \) stands for the price of good 1 in period \( t \) and \( p^2_t \) stands for the price of good 2 in period \( t \).

   The office of national accounts wants to calculate the real GDP growth rate in this economy using the chain index and they hire you to do so. What is the correct growth rate you should report to the office of national accounts?

   (a) 19.7%
   (b) 20.8% \[ \checkmark \]
   (c) 20.1%

2. (6 Points) When we characterize labor supply as an upward sloping relationship between hours worked and wages, we assume that

   (a) the substitution effect dominates the income effect.
   (b) the income effect dominates the substitution effect. \[ \checkmark \]
   (c) the income and the substitution effect cancel out making the relationship between hours worked and wages unambiguous.
3. (6 Points) According to the Solow growth model, which of the following statements is FALSE?

(a) A country which experiences higher population growth than another will have a lower output per worker in steady state.

(b) Steady state consumption in a country which saves more will always be higher than steady state consumption of a country with a lower savings rate. (under the golden ratio)

(c) Capital accumulation alone can not sustain long run growth in capital per worker.

4. (6 Points) Consider a consumer who follows a PIH consumption rule and who is a saver in the first period (i.e., in the first period he consumes less than his current income). Which of the following statements is TRUE?

(a) An increase in the interest rate in the first period will have no effect in the first period's consumption since current consumption only depends on current income.

(b) An increase in the interest rate will make the price of the first period's consumption compared to the second period's consumption more expensive, which means that the first period's consumption will decrease unambiguously.

(c) An increase in the interest rate makes the price of the first period's consumption compared to the second period's consumption more expensive but makes the consumer richer, hence the effect of the first period's consumption after an increase in the interest rate is ambiguous.

5. (6 Points) The relationship between investment and the interest rate will be negative:

(a) Only for firms who finance investment through borrowing.

(b) Because the interest rate determines the opportunity cost of investing, and is a component of the user cost of capital.

(c) Because the interest rate decreases the marginal product of capital making firms want to install less capital.
Consider a PIH consumer, Anna, who receives an income of $4 when she is young and an income of $10 when old. Anna is born with no assets, so \( a = 0 \). The real interest rate that Anna faces for borrowing and saving is equal to \( r \).

1. (7 points) Write down Anna’s intertemporal budget constraint, \( i.e. \) the budget constraint that relates Anna’s lifetime income with Anna’s lifetime consumption. Explain briefly what this budget constraint tells us.

\[
\frac{C_{\text{young}} + C_{\text{old}}}{(1+r)} = 4 + \frac{10}{(1+r)}
\]

This budget constraint tells us that the PV of lifetime consumption equals the PV of lifetime resources.
2. (7 points) Assume that preferences are logarithmic, that is $U(c^y,c^o) = \ln c^y + \beta \ln c^o$, where $\beta \in [0,1]$. Use Anna's optimality condition, $c^o = \beta(1+r)c^y$, and the budget constraint you found in part 1 to find Anna's consumption when young and old as a function of the discount factor, $\beta$, the real interest rate, $r$, the income in period 1 and the income in period 2. Calculate Anna's savings (or borrowing) when young. Under what condition will Anna be a borrower in the first period? Explain briefly how Anna's consumption when young differs from an agent who follows a Keynesian consumption rule, where consumption at time $t$ is equal to $c_t = 0.9Y_t$, where $Y_t$ is income at time $t$.

\[ c^o = \beta(1+r)c^y \]

\[ \frac{c^o}{1+r} + c^y = 4 + \frac{10}{1+r} \]

\[ \frac{\beta(1+r)}{1+r} c^* + c^y = 4 + \frac{10}{1+r} \]

\[ c^y = \frac{4 + \frac{10}{1+r}}{1+\beta} \]

\[ c^* = \frac{c^o}{\beta(1+r)} \]

\[ \frac{c^o}{1+r} + \frac{c^o}{\beta(1+r)} = 4 + \frac{10}{1+r} \]

\[ c^o(1+\beta) = 4(1+r) + 10 \]

\[ c^o = \frac{14 + 4r}{1+\beta} \]

\[ S^y = 4 - c^y, \quad S^o = 10 - c^o + \frac{S^y}{1+r} \]

A Keynesian consumer would consume 3.6 units regardless of future income, whereas Anna will likely consume more to smooth consumption (for large enough $\beta$).
3. (7 points) Suppose that Anna’s income when young increases by $\varepsilon$ but her income when old remains constant. Calculate the increase in Anna’s consumption when young, $\Delta c^y$, and calculate Anna’s marginal propensity to consume when young ($\Delta c^y/\Delta y^y$). Compare this to the marginal propensity to consume of a Keynesian consumer and briefly comment on the differential response of the two consumers to transitory shocks.

$$C_y^* = \frac{4 + \frac{10}{1+r}}{1+\beta} \quad C_y' = \frac{4 + \varepsilon + \frac{10}{1+r}}{1+\beta}$$

$$\Delta C_y^* = \frac{4 + \varepsilon + \frac{10}{1+r}}{1+\beta} - \frac{4 + \frac{10}{1+r}}{1+\beta} = \frac{\varepsilon}{1+\beta}$$

$$\Delta C_y^*_{\text{Keynesian}} = \Delta C_k^* = 0.9\varepsilon$$

$$\frac{\Delta c^y}{\Delta y^y} = \frac{1}{1+\beta}$$

These are the same iff $\frac{1}{1+\beta} = 0.9, \beta = 0.11$.

Since 0.11 means Anna values her old age at only 11% of the utility of her youth, it may be a higher $\beta$ in reality and therefore a larger amount will be saved for the future in the face of transitory income shocks. (Smother consumption)
4. (7 points) Now suppose that Anna faces liquidity constraints, which means Anna can save but is unable to borrow when she is young. Draw the budget constraint in the \(c^y, c^o\) axis. Using the same income stream as above and setting \(\gamma = 0, \beta = 1\), calculate Anna's optimal consumption decision when young. Explain briefly how this compares to the consumption when Anna is young and faces no liquidity constraints.

\[
\begin{align*}
\text{w/o liquidity constraints:} & \\
C^y &= 4 + \frac{10}{1+r} = \frac{14}{2} = 7 \\
C^o &= \frac{14 + 4r}{1 + 1/\beta} = \frac{14}{2} = 7
\end{align*}
\]

\[
\begin{align*}
\text{w/ liquidity constraints} & \\
C^y &= \max\left\{4, \frac{y + \frac{10}{1+r}}{1+\beta}\right\} = 4 \\
C^o &= (y^y - C^y)(1+r) + y^o = 10
\end{align*}
\]
5. (7 points) Suppose as above that Anna faces liquidity constraints when young and her income when young increases by $\varepsilon$ but her income when old remains constant. Calculate the increase in Anna’s consumption when young, $\Delta c^y$ and calculate Anna’s marginal propensity to consume when young ($\Delta c^y/\Delta y^y$). How does Anna’s marginal propensity to consume compares to the marginal propensity to consume of a Keynesian consumer? Comment.

With liquidity constraints Anna will consume every additional dollar under the ideal smoothing value of $\bar{y}$.

That is

$$\Delta c^y = \begin{cases} 
\varepsilon, & y + \varepsilon < \bar{y} \\
\frac{\varepsilon}{12}, & y + \varepsilon \geq \bar{y}
\end{cases}$$

Compared to a Keynesian consumer w/ MPC of $0.9$ this is higher, as every extra marginal dollar is consumed until income has smoothed.
3 Technological Change and the Labor Market (35 points)

In country B there is a firm that produces the unique good of the economy using the following production function:

\[ Y = F(L_s, L_u) = \left( AL_s^{1/2} + L_u^{1/2} \right)^2 \]

where \( A \) is a technological parameter, \( L_s \) is the number of hours of skilled workers the firm hires monthly, and \( L_u \) is the number of hours of unskilled workers the firm hires monthly.

There are two groups of agents in the economy, one group is composed of skilled agents and the other group is composed of unskilled agents. Both groups are equally sized \((N^S = N^U = N)\) and have the same preferences over consumption and leisure which are represented by the following utility function

\[ U(c, l) = \ln c + \eta \ln l \]

where \( \eta \) is a positive constant. The only source of income of the consumers is their wage income, and the two groups of consumers only differ in the wage they receive, which is \( w^S \) for skilled workers and \( w^U \) for unskilled workers. The monthly time endowment of a worker is \( T \). Both the firm and the consumers are price takers, meaning they take the wages, \( w^S, w^U \), and the price of the final good, \( P \), as given.

1. (3 points) Show that the production function satisfies constant returns to scale in the two labor types.

\[
\begin{align*}
Y &= (AL_s^{1/2} + AL_u^{1/2})^2 \\
\lambda Y &= (\lambda AL_s^{1/2} + \lambda AL_u^{1/2})^2 = (\lambda) (AL_s^{1/2} + AL_u^{1/2})^2 \\
\lambda Y &= \lambda (AL_s^{1/2} + AL_u^{1/2})^2 \\
\therefore \text{ constant returns to scale.}
\end{align*}
\]
2. (5 points) Using the firm’s optimality condition we have seen in class, and using the fact that the marginal product of worker of type \( s \) is

\[
AL_s^{-1/2} \left( AL_s^{1/2} + L_u^{1/2} \right)
\]

and the marginal product of labor of a worker of type \( u \) is

\[
L_u^{-1/2} \left( AL_s^{1/2} + L_u^{1/2} \right),
\]

find the relative demand of skilled workers to unskilled workers \( (L^S/L^U) \) as a function of the relative wage \( (w^S/w^U) \) and the technological parameter.

\[
\text{Labor demanded s.t.}
\]

\[
MPN = \frac{\text{Wage}}{P}
\]

\[
AL_s^{-1/2} \left( AL_s^{1/2} + L_u^{1/2} \right) = \frac{w^s}{P}
\]

\[
L_u^{-1/2} \left( AL_s^{1/2} + L_u^{1/2} \right) = \frac{w^u}{P}
\]

\[
\frac{AL_s^{-1/2} \left( AL_s^{1/2} + L_u^{1/2} \right)}{L_u^{-1/2} \left( AL_s^{1/2} + L_u^{1/2} \right)} = \frac{w^s}{w^u}
\]

\[
\frac{\sqrt{L_u}}{\sqrt{L_s}} = \left( \frac{w^s}{w^u} \right)^2
\]

\[
\frac{L_s}{L_u} = \frac{A^2}{(w^s/w^u)^2}
\]
3. (4 points) Using the time constraint $T = n + l$, where $n$ is number of hours worked, write the budget constraint of a consumer of type $s$ and the budget constraint of a consumer of type $u$ as a function of leisure, consumption, the wage rate of the type, the price $P$, and $T$.

$$u(n, l) = \ln \left( \frac{n \cdot w}{p} \right) + \gamma \ln l$$

$$T = \frac{c \cdot P}{\omega_{u/s}} + l$$

$$c = e, \quad n = \frac{c \cdot P}{\omega}$$
4. (5 points) Using the time constraint \( T = n + l \), the budget constraint, and the fact the marginal utility of consumption and the marginal utility of leisure are given by

\[
MU(c) = \frac{1}{c} \\
MU(l) = \frac{\eta}{l},
\]

find the optimal labor supply of individuals and the optimal demand of the final good for skilled and unskilled consumers.

\[
T = \frac{c \cdot P}{\omega_{w/s}} + l
\]

\[
\frac{MU(c)}{MU(l)} \text{ s.t. } MU(c) = MU(l) \\
\frac{1}{c} = \frac{\eta}{l}
\]

\[
l = \eta c
\]

\[
T = \frac{c \cdot P}{\omega_{w/s}} + \eta c
\]

\[
T = n + \eta \left( \frac{w}{P} \right) n
\]

\[
\frac{T}{1 + \eta \left( \frac{w}{P} \right)} = n
\]

**Optimal Demand of Goods:**

\[
D = \frac{T}{P/w_s + \eta} + \frac{T}{P/w_u + \eta}
\]

**Optimal Supply of Labor:**

\[
N \frac{T}{1 + \eta \left( \frac{w_s}{P} \right)} \text{ skilled} \\
N \frac{T}{1 + \eta \left( \frac{w_u}{P} \right)} \text{ unskilled}
\]

\[
h = \frac{1}{1 + \eta} T
\]
5. (5 points) Using the labor supplies for the two groups of consumer and the relative labor demand of the firm, find the equilibrium relative wage \( \left( \frac{w^S}{w^U} \right) \).

\[
A = \frac{\frac{A}{L} \cdot L}{\sqrt{L}} \left( \frac{1 + \frac{w^S}{P}}{1 + \frac{w^U}{P}} \right)^{\frac{1}{2}}
\]

\[
\frac{NT}{1 + \eta \left( \frac{w^S}{P} \right)} = \frac{A^2}{\left( \frac{w^S}{w^U} \right)^2}
\]

\[
\frac{1 + \eta \left( \frac{w^U}{P} \right)}{1 + \eta \left( \frac{w^S}{P} \right)} = \frac{A^2}{\left( \frac{w^U}{w^S} \right)^2}
\]

\[
\left( 1 + \eta \left( \frac{w^U}{P} \right) \right) \left( \frac{w^S}{w^U} \right)^2 = \chi
\]

\[
\frac{w^S}{w^U} = A
\]
6. (5 points) What is the effect of a technological increase (an increase in $A$) in the wage gap between skilled and unskilled workers? Explain briefly what happens to the labor demand of the two types of workers.

An increase in $A$ has an increasing effect on the wage gap because

\[
\frac{1}{L_s}W_s = \left( \frac{A\sqrt{L_u}}{L_s} \right) W_u
\]

\[
W_s' = \left( \frac{(2A)\sqrt{L_u}}{JL_s} \right) W_u
\]

\[
W_s = 2W_u
\]

Labor demand for skilled workers will increase and demand for unskilled workers decreases.
7. (4 points) Now imagine that as a result of a national trend, more and more people go to college so the ratio of skilled to unskilled workers rises (a rise in $N^s/N^u$). What will happen to the relative wage between skilled and unskilled workers as a result of this national trend?

The relative wage difference will decrease as the supply of labor for skilled work increases.
8. (4 points) Finally assume that the current technological level, $A$, is endogenous and is a function of the ratio between skilled and unskilled workers. In particular, assume $A = (N^s/N^u)\beta$, $\beta > 0$. What will happen to the wage ratio as a result of the national trend mentioned before?

If $\beta$ is high enough the effect of productivity will increase demand faster than the increase in supply (1) so the wage gap will increase.
**Quiz 2**  
**Thursday, November 5\textsuperscript{th}**  
**7:30 PM – 9 PM**

Please answer the following questions. Write your answers directly on the quiz. You can achieve a total of 100 points. There are 5 multiple-choice questions, followed by 2 free response questions. You should read all of the questions first.  

*Good luck!*

**NAME:**  

Scott Young

**MIT ID NUMBER:**

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1 Multiple choice (30 points)

1. (6 points) The increase in use of ATMs decreases the currency/deposit ratio ($c_d$). According to the Keynesian theory of sticky prices:
   (a) output increases and the interest rate goes down,
   (b) output increases and the interest rate goes up,
   (c) output decreases and the interest rate goes up.

2. (6 points) In the standard IS-LM model, an increase in Government spending ($G$) without changing taxes has
   (a) a positive effect on equilibrium consumption,
   (b) a negative effect on equilibrium consumption,
   (c) an ambiguous effect on equilibrium consumption.

3. (6 points) According to the misperception theory, an expected increase in money supply
   (a) increases output and increases interest rate,
   (b) increases output and decreases interest rate,
   (c) has an effect on neither output nor interest rates.

4. (6 points) Consider a fractional reserve banking system with a legally required reserve-deposit ratio of $m$. Suppose that an individual deposits $ID$ dollars in one bank. Then, the economy-wide change in total deposits
   (a) will be at most $ID / m$,
   (b) will be equal to $ID / m$,
   (c) will be equal to $m \cdot ID$.

5. (6 points) According to the Taylor rule, a positive output gap (i.e. real GDP above potential real GDP) will most likely result in
   (a) the Fed adjusting its estimate of potential real GDP,
   (b) the Fed decreasing the nominal federal funds rate,
   (c) the Fed increasing the nominal federal funds rate.
2 IS-LM with Liquidity Trap (35 points)

Consider the following IS-LM model with prices fixed at \( P = 1 \) (we are in the short run):

\[
\frac{M^d}{P} = Y - r \\
C = 1 + 0.5Y \\
I = 1 - 0.5r \\
G = \overline{G} \\
Y = C + I + G \\
\frac{M^s}{P} = \frac{\overline{M}}{\overline{P}} \\
\frac{M^d}{P} \leq \frac{M^s}{P}, \text{ with } \frac{M^d}{P} = \frac{M^s}{P} \text{ if } r > 0 \\
r = i - \pi^e \\
\pi^e = 0
\]

1. (7 points) Explain the minimum value that the real interest rate, \( r \), can take.

The real interest rate can be a minimum of \(-\pi^e\) or in this case, 0 (nominal rates cannot be lower than zero!)

2. (7 points) Derive the IS curve.

\[
I = 1 - 0.5r \\
S = Y - C - \overline{G} \\
S = Y - (1 + 0.5Y) - \overline{G} \\
S = 0.5Y - 1 - \overline{G}
\]

\[
1 - 0.5r = 0.5Y - 1 - \overline{G} \\
Y = \frac{4 - r + 2\overline{G}}{0.5}
\]
3. (7 points) Write down the LM curve.

\[ \frac{\bar{M}}{\bar{P}} = Y - r \]

\[ Y = \frac{\bar{M}}{\bar{P}} + r \]

4. (7 points) What are the equilibrium interest rate and output level in the economy? What is the condition for the equilibrium interest rate to be positive?

\[ Y = \frac{\bar{M}}{\bar{P}} + r \]

\[ Y = 4 - r + 2\bar{G} \]

\[ 4 - r + 2\bar{G} = \frac{\bar{M}}{\bar{P}} + 2r \]

\[ r = \frac{4 + 2\bar{G} - \bar{M}}{2} \]

\[ Y^* = \frac{\bar{M}}{\bar{P}} + 2 + \bar{G} - \frac{\bar{M}}{\bar{P}} = 2 + \bar{G} \]

The interest rate will be positive if

\[ \rho(2 + \bar{G}) > \frac{\bar{M}}{\bar{P}} \]
5. (7 points) Suppose that the economy described above is going through a recession and the government is trying to stimulate the economy. When will monetary policy be effective in stimulating the economy? Explain why under certain conditions monetary policy fails to be effective as a policy instrument.

Monetary policy will be effective so long as interest rates are above zero. Since central banks cannot lower it below zero, policy options are limited/ineffective while in such a liquidity trap.
3 AS-AD (35 points)

[Supply side] Consider a labor market characterized by the following production and labor supply functions:

\[ F(N) = 20N - N^2 \]
\[ N^* = \frac{1}{2} \frac{w}{P} \]

1. (3 points) Using the fact that the marginal product of labor \( MPN = 20 - 2N \), obtain and graph the labor demand function.

\[ \text{Labor demand s.t.} \]
\[ MPN = \frac{w}{P} \]
\[ 20 - 2N = 2N \]
\[ 4N = 20 \]
\[ N^d = 5 \]

2. (3 points) Graph the labor supply function and solve for equilibrium (find \( \left( \frac{w}{P} \right)^* \) and \( N^* \)) in the labor market.

---

\[ 0 = \left( \frac{w}{P} \right)^* \]
\[ (20 - 2N) = MPN \]
\[ \text{slope} = \frac{w}{P} \]
\[ \text{Labor supply} \]
\[ N = N^* \]
3. (5 points) Take prices \( p \) as given. Consider the case in which the government introduces a minimum nominal wage \( \bar{w} = 50 \). Explain in words the qualitative effects of this policy on equilibrium wages and employment as a function of \( p \). (Hint: consider three cases \( p \geq 5, \frac{5}{2} < p < 5 \) and \( p \leq \frac{5}{2} \)).

CASE 1: \( p \geq 5 \)

\[
\left( \frac{w}{P} \right)^* = 10
\]

\[
\frac{50}{5} > 10
\]

No impact, wages are already above minimum

CASE 2: \( \frac{5}{2} < p < 5 \)

\[
N^* = \frac{50}{2p}
\]

\[
M_F N = 20 - \frac{50}{p}
\]

\[
20 - \frac{50}{p} > 0\quad \text{S.t. } p > \frac{5}{2}
\]

Employment will drop, but need to zero

CASE 3: \( p \leq \frac{5}{2} \)

\[
20 - \frac{50}{p} \leq 0
\]

Employment will cease.

4. (5 points) Solve for the real wage and number of workers, taking prices as given. Make sure you consider the three cases \( p \geq 5, \frac{5}{2} < p < 5 \) and \( p \leq \frac{5}{2} \).

CASE 1: Real wage = \( \left( \frac{w}{P} \right)^* = 10 \)

\[
No. \ N = 5
\]

CASE 2: \( \left( \frac{w}{P} \right)^* = \frac{50}{P} \)

\[
20 - 2N = \frac{50}{20} \quad N = \frac{20 - 50}{20}
\]

CASE 3: None (no employees)
5. (4 points) Solve for the aggregate supply function and graph it.

\[ \text{Long Run} \]

\[ AS = Y^* + 75 \quad (N = N^*) \]

\[ \text{Short Run} \]

\[ AS = p = p \]

[Demand side] Consider the demand side characterized by the following consumption, investment and real money balances demand functions. Also, government expenditures are \( g = 50 \).

\[ c = 25 + \frac{y}{2} \]
\[ i = 25 - \frac{r}{2} \]
\[ \frac{M^d}{p} = 100 - \frac{r}{2} + \frac{y}{2} \]

6. (4 points) Obtain the IS and LM relations as a function of money supply \( M^s \).

\[ i = Y - C - G = 25 - \frac{r}{2} \]
\[ \frac{y}{2} - (25 + \frac{y}{2}) - 50 = 25 - \frac{r}{2} \]
\[ 0.5y - 25 - 50 = 25 - \frac{r}{2} \]
\[ y = 200 - r \quad \text{IS} \]

\[ \frac{M^s}{p} = 100 - \frac{r}{2} + \frac{y}{2} \]
\[ Y = \frac{2M^d}{p} + r + 200 \quad \text{LM} \]
7. (3 points) Obtain the aggregate demand function.

\[ 200 - r = y \quad \Rightarrow \quad r = 200 - y \]

\[ y = \frac{2M^s}{P} + r - 200 \quad \Rightarrow \quad r = y + 200 - \frac{2M^s}{P} \]

\[ 200 - y = y + 200 - \frac{2M^s}{P} \]

\[ \frac{2M^s}{P} \]

\[ y = M^s \cdot \left( \frac{1}{p} \right) \]

[Equilibrium]

8. (4 points) For \( M^s = 150 \), solve for equilibrium output and prices. What is the effect on output of an expansionary monetary policy?

\[ y = \frac{150}{P} \]

\[ 75 = \frac{150}{P} \]

\[ p^* = 2, \quad y = y^* = 75 \]

An expansionary policy in LR has no effect on output, in SR, it increases \( y \) and prices.
9. (4 points) For $M^* = 450$, solve for equilibrium output and prices. What is the effect on output of an expansionary monetary policy?

$$75 = \frac{450}{p}$$

$$p = 6$$

In LR, again no difference.
In SR, prices and $Y$ rise.
14.02 Principles of Macroeconomics
Fall 2009

Quiz 3
Thursday, December 3rd
7:30 PM – 9 PM

Please answer the following questions. Write your answers directly on the quiz. You can achieve a total of 100 points. There are 5 multiple-choice questions, followed by 2 free response questions. You should read all of the questions first.

Good luck!

NAME: Scott Young

MIT ID NUMBER: ________________________________

TA: ________________________________

CLASS TIME: ________________________________

EMAIL: ________________________________

(Table is for corrector use only)

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1 Multiple Choice (30 points)

1. [6 points] An increase in domestic prices:
   (a) Depreciates the domestic currency.
   (b) Appreciates the domestic currency.
   (c) Has an ambiguous effect on the domestic currency.

2. [6 points] In the short run, a contractionary monetary policy in an open economy...
   (a) Contracts output and generates a depreciation.
   (b) Contracts output and generates an appreciation.
   (c) Expands output and generates a depreciation.

3. [6 points] There is a 30% tax levied on income above $15,000. A person with income of $18,000 faces:
   (a) A marginal tax rate of 30% and an average tax rate of 30%.
   (b) A marginal tax rate of 30% and an average tax rate of 25%.
   (c) A marginal tax rate of 30% and an average tax rate of 5%.

4. [6 points] Consider a 2 country world, countries A and B. If under autarky the interest rates satisfy \( r_A > r_B \), after integration:
   (a) Savings and investment in country A decrease.
   (b) Savings decrease in country A and country B.
   (c) Savings in country A and investment in country B decrease.

5. [6 points] Deposit insurance...
   (a) Is a successful tool in preventing bank runs but it is costly for the government.
   (b) Is a successful tool in preventing bank runs and it may not be costly for the government.
   (c) Has not proven to be a successful tool in preventing bank runs.
2 Barro 1979 (35 points)

Consider the 2 period consumption model. Denote \( c_t \) consumption and \( y_t \) income at time \( t = 1, 2 \). Consumers and government may save and borrow at period \( t = 1 \) at a given interest rate \( r \). Each period, the government collects lump sum taxes \( t_t \) and government expenditures are \( g_t \). Income, taxes and government expenditures are exogenous for the consumer. Utility for consumers is given by \( U(c_1, c_2) = u(c_1) + \beta u(c_2) \). Assume \( \beta (1 + r) = 1 \).

1. [2 points] Obtain the intertemporal budget constraint for the consumer.

\[
c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} - \left( t_1 + \frac{t_2}{1+r} \right)
\]

2. [2 points] Obtain the intertemporal budget constraint for the government.

\[
g_1 + \frac{g_2}{1+r} = t_1 + \frac{t_2}{1+r}
\]
3. [6 points] Combine the previous expressions to sketch a proof of the statement "The timing of taxes does not affect consumption choices as long as the present value of consumption is not affected".

\[ c_1 + \frac{c_2}{1+r} = PV_c \]

\[ y_1 + \frac{y_2}{1+r} = PV_y \]

\[ t_1 + \frac{t_2}{1+r} = PV_t \]

\[ PV_c = PV_y - PV_t \]

The equation under max. utility is valid.

If this remains constant, \( c_1 \) don't depend on \( t_1, t_2 \).

4. Let us consider the case in which taxation is no longer a one-to-one transfer of purchasing power from consumers to the government. In particular, assume that when taxes \( t_t \) are collected, an amount \( \gamma(t_t) - b t_t^2 \) of income is wasted, where \( b \) is a positive constant. We may think of this as collection costs or "deadweight losses." The marginal loss given by \( \gamma'(t) = 2bt \) is increasing in \( t \).

(a) [3 points] Obtain the intertemporal budget constraint for the consumer.

\[ PV_c = PV_y - PV_t \]

\[ \frac{c_1 + c_2}{1+r} = y_1 + \frac{y_2}{1+r} - \left( t_1 + \frac{t_2}{1+r} \right) \]

The budget constraint is the same however \( t_t \) will increase.
(b) [2 points] Obtain the intertemporal budget constraint for the government.

\[ g_1 + \frac{g_2}{1+r} = t_1 - bt_1^2 + \frac{t_2}{r+1} - b \left( \frac{t_2}{r+1} \right)^2 \]

(c) [5 points] Does the Ricardian equivalence still hold? Explain.

No, because loss increases with \( t \), the discount factor \( 1+r \) is squared for the loss, meaning taxes paid in lump sums will reduce PVLR for consumers.
(d) [4 points] Suppose that the consumer could choose the timing of taxes subject to the intertemporal budget constraint for the government. Recall that under the assumption $\beta(1+r) = 1, c_1 = c_2$. State the maximization problem to determine the optimal choice of $t_1$ and $t_2$. No need to solve.

\[
\text{we want to maximize revenue and minimize loss. So we can solve the dual problem:}
\]

\[
\text{PV of loss} = bt_1^2 + b \left( \frac{t_2}{1+r} \right)^2 = \text{minimize}
\]

(e) [2 points] Interpret the following two conditions that determine optimal tax collection.

\[
2bt_1^* = 2bt_2^*
\]

\[
g_1 + \frac{g_2}{1+r} = t_1^* + \frac{t_2^*}{1+r}
\]

The first states that marginal loss for taxes in both time periods must be equal. The second states that PV of tax collections must equal PV of government expenses.
(f) [4 points] Solve for the optimal tax path.

\[ t_1 = t_2 = t \]

\[ g_1 + \frac{g_2}{1+r} = t + \frac{t}{1+r} = t \left( 1 + \frac{1}{1+r} \right) \]

\[ t = \frac{g_1 + \frac{g_2}{1+r}}{1 + \frac{1}{1+r}} = \text{tax collected in each term} \]

(g) [5 points] Use your results to justify how a government should finance a temporary increase in government expenditures (e.g., wartime).

"Governments should try to smooth taxation, therefore financing temporary increases via budget deficits."
3 Open Economy (35 points)

Consider a world economy with two countries. The home country is characterized by:

\[
\begin{align*}
C &= c_0 + c_1 Y \\
I &= c_2 \\
X &= d_1 \bar{Y} \\
M/e &= d_2 Y \\
\end{align*}
\]

where \( C, Y, \bar{Y}, I, M, X, e \) denote aggregate consumption, domestic output, foreign output, investment, imports, exports and the real exchange rate, respectively. Assume \( c_0, c_2 > 0, 0 \leq c_1 \leq 1, 0 \leq d_1 \leq 1, 1 - c_1 + m > 0, 0 \leq d_2 \leq 1. \)

The foreign country is characterized by

\[
\begin{align*}
\tilde{C} &= \tilde{c}_0 + \tilde{c}_1 \bar{Y} \\
\tilde{I} &= \tilde{c}_2 \\
\tilde{M}/\tilde{e} &= \tilde{d}_1 \bar{Y} \\
\tilde{X} &= \frac{\tilde{d}_2 Y}{\tilde{e}} \\
\end{align*}
\]

Assume that parameters in the foreign economy satisfy assumptions analogous to the ones for the domestic economy.

We will assume throughout the question that the real exchange rate (\( e \)) is exogenously given.

1. [5 points] Find the equilibrium level of output in the domestic goods market as a function of \( e, C, \bar{Y} \) and parameters. Find the multiplier. Find equilibrium in foreign goods market as a function of \( \tilde{e}, \tilde{G}, \bar{Y} \) and parameters.

\[
\begin{align*}
Y &= C + I + G + X \\
\bar{Y} &= c_0 + c_1 Y + c_2 + G + \frac{d_1 \bar{Y}}{e} \\
Y (1 - c_1) &= c_0 + c_2 + G + \frac{d_1 \bar{Y}}{e} \\
Y &= \frac{c_0 + c_2 + G + \frac{d_1 \bar{Y}}{e}}{1 - c_1} \\
\end{align*}
\]

\[
\begin{align*}
\bar{Y} &= \frac{\tilde{c}_0 + \tilde{c}_1 \bar{Y} + \bar{G} + \frac{\tilde{d}_2 Y}{\tilde{e}}}{1 - \tilde{c}_1} \\
\end{align*}
\]
2. [3 points] How do net exports depend on the real exchange rate in this model?

Net exports are reduced fractionally by increases in e.

\[ X \propto \frac{1}{e} \]

3. [3 points] State a condition relating e and \( \hat{e} \).

\[ X = -X \leftarrow \text{net exports for one country must equal net imports for other} \]

\[ \frac{d_1 Y}{e} = -\frac{d_2 Y}{\hat{e}} \]

\[ \frac{\hat{e}}{e} = -\frac{d_2 Y}{d_1 Y} \]

\[ e = \frac{1}{\hat{e}} \]
4. [6 points] Solve for domestic output as a function of $c, G, \bar{G}$ and parameters.

$$Y = c_0 + c_1 + G + \frac{d_1 \left( c_0 + c_1 + \bar{G} + d_1 \right)}{(1 - c_1)e}$$

$$Y(1 - c_1) = c_0 + c_2 + G + \frac{d_1 d_2 Y}{\bar{e} e (1 - c_1)} + \frac{d_1 (c_0 + c_1 + \bar{G})}{(1 - c_1)e}$$

$$Y(1 - c_1 - \frac{d_1 d_2}{\bar{e} e (1 - c_1)}) = c_0 + c_2 + G + \frac{d_1 (c_0 + c_1 + \bar{G})}{(1 - c_1)e}$$

5. [6 points] How is domestic output affected by a real depreciation of the home goods relative to the foreign goods? What about foreign output? Explain.

Domestic output will increase as exports grow.
Foreign output will drop.
6. [6 points ] Suppose that each country may either "not act" or "attempt to depreciate." If a country attempts to depreciate and the other country does not act, then the depreciation is achieved. However, if both countries attempt to depreciate, there is no change in the real exchange rate. Show that each country will choose to "attempt to depreciate".

\[ b/c \ Y \text{ increases on "attempt to depreciate" for both sides, and } Y \text{ remains constant if both don't act, and lowers if one fails to depreciate but the other does, the strategy "attempt to depreciate" is dominant for both countries.} \]

7. [6 points] In light of the previous result, interpret the following paragraph of the article "Currency war fears after Swiss devaluation" (12th March 2009)

Analysts have warned of the emerging threat of currency wars following a Swiss market intervention. The Swiss franc fell by as much as 3.2 per cent against the euro to 1.53 - the biggest decline since 1999 - after the country's central bank said it was taking deliberate measures to weaken its currency. The move raises fears of retaliatory measures by other countries seeking to give their exporters a competitive edge.

[Hint: if prices are sticky, the real and nominal exchange rates move together]

As stated previously, other countries will try to force a currency depreciation to spike exports (or at least avoid a flood of cheap imports).