18.02 - Multivariate Calculus

Scott Young   Exam written at 12:00 pm
October 14th, 2011

EVALUATION

	SCORE	TOTAL
Q1	25	25
Q2	10	15
Q3	20	20
Q4	20	20
Q5	20	20
Q6	20	20
Q7	~~16~~ 12	20
Q8	0	20
Q9	12	20
Q10	11	25
Q11	21	25
Q12	20	20
Q13	0	20

$\frac{191}{270}$

71.1°

SCOTT'S NOTE ON GRADING:
WHEN AN ANSWER IS INCORRECT, PART MARKS ARE GIVEN, -4 FOR ALGERRAIC, ARITHMATIC OR NON-ESSENTIAL ERROR (MEANING NOT A FAULT IN THE METHODS OF THE CLASS ITSELF). -8 FOR ALGERRAIC OR SIMPLE ERRORS WHICH DRASTI SIMPLIFY THE PROBLEM (OR OTHER). OTHER CONCEPT ERRORS AWARDED ZERO - PART MARKS WITH DISCRETION.

180 ~ 12 min/

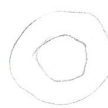
1) For each of (a)-(e) below: If the statement is true, write TRUE. If the statement is false, write FALSE. (Please do not use the abbreviations T and F.) No explanations are required in this problem.

(a) (5 pts.) If $f(x,y)$ is a continuously differentiable function on $\mathbb{R}^2$, and $\overset{f_{xx}}{\dfrac{\partial^2 f}{\partial x^2}} = 0$ at every point of $\mathbb{R}^2$, then there exist constants a and b such that $f(x,y) = ax + b$ for all x and y.

✓ FALSE

(b) (5 pts.) The annulus in $\mathbb{R}^2$ defined by $9 \leq x^2 + y^2 \leq 16$ is simply connected.

✓ FALSE



(c) (5 pts.) If $f(x,y)$ is a function whose second derivatives exist and are continuous everywhere on $\mathbb{R}^2$, and

$$f(0,0) = f_x(0,0) = f_y(0,0) = f_{xx}(0,0) = f_{yy}(0,0) = 0$$

and $f_{xy}(0,0) \neq 0$, then f has a saddle point at $(0,0)$.

✓ TRUE

(d) (5 pts.) If A is a 3×3 matrix, and $\mathbf{b}$ is a column vector in $\mathbb{R}^3$, and the square system $A\mathbf{x} = \mathbf{b}$ has more than one solution $\mathbf{x}$, then $\det A = 0$.

✓ TRUE

(e) (5 pts.) If $\mathbf{F}$ is a continuously differentiable 3D vector field on $\mathbb{R}^3$ such that $\operatorname{curl} \mathbf{F} = \mathbf{0}$ everywhere, and A and B are two points in $\mathbb{R}^3$, then the value of $\int_C \mathbf{F} \cdot d\mathbf{r}$ is the same for every piecewise smooth path C from A to B.

✓ TRUE

$\dfrac{25}{25}$

2) (a) (5 pts.) For which pairs of real numbers (a, b) does the matrix

$$A = \begin{pmatrix} a & b & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

have an inverse?

$\boxed{\begin{matrix} a=0 \\ b=1 \end{matrix}}$

$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ ← has a solution

✓

x $\boxed{0}$ ← Missed all possible solutions

(b) (10 pts.) For such pairs (a, b), compute A^{-1}.
(Its entries may depend on a and b. *Suggestion:* Once you have the answer, check it!)

$$A^{-1} = \begin{bmatrix} +\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} & -\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} & +\begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} \\ -\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} & +\begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \\ +\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} & +\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

CARRY FORWARD ERROR

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$\det(A) = 1$

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ ✓

✓

3) Let L be the line that passes through $(7, 3, -1)$ and is perpendicular to the plane P with equation $2x + y - z = 6$. Find the point where L intersects P.

L is a line passing through (7,3,-1) and normal to 2x+y-z=6, find intersection

$\vec{N} = \langle 2, 1, -1 \rangle$

$$\text{line} = \begin{cases} x = 2t+7 \\ y = t+3 \\ z = -t-1 \end{cases}$$

$$2(2t+7) + (t+3) - (-t-1) = 6$$

$$4t + 14 + t + 3 + t + 1 = 6$$

$$6t + 18 = 6$$

$$6t = -12$$

$t = -2$ @ intersection point

$\therefore$

$$x = 2(-2)+7 = 3$$
$$y = (-2)+3 = 1$$
$$z = -(-2)-1 = 1$$

$\boxed{\text{intersect at point } (3,1,1)}$ ✓

CONFIRM

$2(3)+(1)-(1)=6$ ✓

$\boxed{20}$

4) A particle is moving in the plane so that its distance from the origin is increasing at a constant rate of 2 meters per second, and its argument θ is increasing at a rate of 3 radians per second. At a time when the particle is at $(4, 3)$, what is its velocity vector?

$$\frac{dr}{dt} = 2$$

$$\frac{d\theta}{dt} = 3$$

what is $\vec{v}$ or $\frac{d\vec{r}}{dt}$?

at $x = 4$
$y = 3$

$x = r\cos\theta$
$y = r\sin\theta$

$dx = dr\cos\theta - r\sin\theta\, d\theta$
$dy = dr\sin\theta + r\cos\theta\, d\theta$ $\Rightarrow$

$\frac{dx}{dt} = \frac{dr}{dt}\cos\theta - r\sin$
$\frac{dy}{dt} = \frac{dr}{dt}\sin\theta + r\cos\theta$

$\vec{v} = \langle \frac{dx}{dt}, \frac{dy}{dt} \rangle$

$\frac{dx}{dt} = 2\cos\theta - r\sin\theta \cdot 3$
$= 2\cos(\tan^{-1}(\frac{3}{4})) - 5\sin(\tan^{-1}(\frac{3}{4}))$
$= \frac{8}{5} - 15 \cdot \frac{3}{5} = \frac{8}{5} - 9$

$\frac{dy}{dt} = 2\sin\theta + r\cos\theta \cdot 3$

$\frac{dy}{dt} = 2\sin(\tan^{-1}(\frac{3}{4})) + 5 \cdot \cos(\tan^{-1}(\frac{3}{4})) \cdot 3$

$= \frac{6}{5} + 12$

$4 = r\cos\theta$
$3 = r\sin\theta$

$r = \frac{4}{\cos\theta}$

$3 = \frac{4\sin\theta}{\cos\theta}$

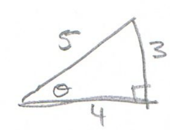
$3 = 4\tan\theta$

$\frac{3}{4} = \tan\theta$

$\boxed{\theta = \tan^{-1}\left(\frac{3}{4}\right)}$

$4 = r\cos(\tan$

$r = \frac{4}{\cos(\tan^{-1}(}$



$r = \frac{4}{4/5}$

$\boxed{r = 5}$

$$\boxed{\vec{v} = \langle \frac{8}{5} - 9, \frac{6}{5} + 12 \rangle}$$

✓

20

5) Let $f(x,y) = x^2 - xy + y^6$. Find the *minimum* value of the directional derivative $D_{\mathbf{u}}f$ at the point $(2,1)$ as $\mathbf{u}$ varies over unit vectors in the plane.

Find minimum of $D_u f$ at pt $(2,1)$

$-\nabla f$ = minimum

$= \langle -2x+y,\ x-6y^5 \rangle$

$= \langle -2(2)+1,\ (2)-6(1)^5 \rangle$

$= \langle -3, -4 \rangle$

$\nabla f = \langle f_x, f_y \rangle$

$\nabla f = \langle 2x-y,\ 6y^5-x \rangle$

$\left.\frac{\partial f}{\partial s}\right|_{\hat{u}} = \nabla f \cdot \hat{u} = (2x-y)(u_x) + (6y^5-x)(u_y)$

Let $\hat{u} = \langle u_x, u_y \rangle$

x-component of u, not $\frac{\partial u}{\partial x}$

$g(2,1) = (2(2)-(1))u_x + (6(1)^5-(2))u_y$

$u = \langle \frac{-3}{4}, \frac{\sqrt{7}}{4} \rangle$

$\boxed{= \cancel{3u_x + 4u_y}}$

OMIT

~~minimum value of is when~~

~~min $g = 3u_x + 4u_y$~~

~~and $u_x^2 + u_y^2 = 1$ ← vector size~~

~~$u_x = (1-u_y^2)^{1/2}$~~

~~$g = 3(1-u_y^2)^{1/2} + 4u_y$~~

~~$0 = \frac{dg}{du_y} = \frac{3\cdot(-2u_y)}{2\sqrt{1-u_y^2}} + 4$~~

~~$\frac{-3}{\sqrt{1-u_y^2}} = -4$~~

~~$\left(\frac{3}{4}\right)^2 = 1 - u_y^2$~~

~~$\frac{7}{16} = u_y^2$~~

~~$\frac{7}{16} + u_x^2 = 1$~~

~~$u_x^2 = \frac{9}{16}$~~

~~$u_x = \pm\frac{3}{4}$~~

~~$\boxed{u_y = \pm\frac{\sqrt{7}}{4}}$~~

$= 3\left(\frac{-3}{4}\right) + 4\left(\frac{\sqrt{7}}{4}\right)$

$= \frac{-9}{4} + \sqrt{7}$

$= 3\left(\frac{3}{4}\right) + \sqrt{7}$

$= \frac{9}{4} + \sqrt{7}$

ON BACK

Minimum directional derivative

in $\langle -3, -4 \rangle$

$\hat{u} = \langle -\frac{3}{5}, -\frac{4}{5} \rangle$

$\nabla f \cdot u = \langle 2x - y, 6y^5 - x \rangle \cdot \langle -\frac{3}{5}, -\frac{4}{5} \rangle$

$\cancel{-\frac{6}{5}(x) + \frac{3}{5}(y) + 6y^5 \cdot \frac{4}{5} + \frac{x4}{5}}$

$\langle 3, 4 \rangle \cdot \langle -\frac{3}{5}, -\frac{4}{5} \rangle$

$= -\frac{9}{5} - \frac{16}{5} = \boxed{-5}$ ✓

$\boxed{20}$

6) Find an equation for the tangent plane to the surface $x^2 + y^2 + 3z = 8$ at the point $(2, 1, 1)$.

$\vec{N} = \langle f_x, f_y, f_z \rangle$ $\vec{N} =$

$= \langle 2x_0, 2y_0, 3 \rangle$

a b c

Normal vector

$(2x_0)x + (2y_0)y + 3z = d$

$\boxed{4x + 2y + 3z = 13}$ ✓

$4(2) + 2(1) + 3(1) = 13$

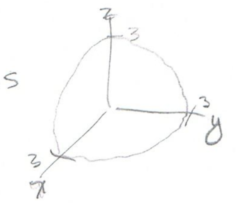
20

7) Let S be the part of the sphere $x^2+y^2+z^2=9$ where x, y, z are all positive. Find the minimum *value* of the function

$$\frac{8}{x}+\frac{8}{y}+\frac{1}{z}$$

on S, or explain why it does not exist.

S is



find the minimum of

$$\frac{8}{x}+\frac{8}{y}+\frac{1}{z}$$

$$f=\frac{8}{x}+\frac{8}{y}+\frac{1}{z}$$

$$g=x^2+y^2+z^2-9$$

$$\nabla f=\lambda\nabla g$$

The two gradients are equivalent b/c the gradients are tangenti at a minimum

$$f_x=g_x\lambda \qquad -\frac{8}{x^2}=2x\cdot\lambda$$

$$f_y=g_y\lambda \qquad -\frac{8}{y^2}=2y\lambda$$

$$f_z=g_z\lambda \qquad -\frac{1}{z^2}=2z\lambda$$

$$x^2+y^2+z^2-9=0$$

4 eqtns ✓

$$-\frac{8}{2}=\frac{2x^3\lambda}{2}$$

$$-4=x^3\lambda$$

$$x=\sqrt[3]{\frac{-4}{\lambda}}$$

$$y=\sqrt[3]{\frac{-4}{\lambda}}$$

$$z=\sqrt[3]{\frac{-1}{\lambda}}$$

FORGOT TO EVALUATE VALUE −4

$$\left(\frac{-4}{\lambda}\right)^{2/3}+\left(\frac{-4}{\lambda}\right)^{2/3}+\left(\frac{-1}{\lambda}\right)^{2/3}=9$$

$$2\left(\frac{1}{\lambda}\right)^{2/3}+2\left(\frac{1}{\lambda}\right)^{2/3}+1\left(\frac{1}{\lambda}\right)^{2/3}=9$$

$$5\left(\frac{1}{\lambda}\right)^{2/3}=9$$

$$\frac{1}{\lambda}=\left(\frac{9}{5}\right)^{3/2}$$

$$\frac{1}{\lambda}=\frac{27}{5\sqrt{5}}$$

$$\lambda=\frac{5\sqrt{5}}{27}$$

ARITHMATIC ERROR −4

$$x=\sqrt[3]{-4\cdot\left(\frac{27}{5\sqrt{5}}\right)}$$

$$x=\sqrt[3]{\frac{-108}{5\sqrt{5}}}$$

$$y=\sqrt[3]{\frac{-108}{5\sqrt{5}}}$$

$$z=\sqrt[3]{\frac{-27}{5\sqrt{5}}}$$

$$=-3\sqrt[3]{\frac{1}{5\sqrt{5}}}$$

8) Suppose that $s(x,y)$ and $t(x,y)$ are differentiable functions such that it is possible to express x as a differentiable function of s and t. Express $\left(\frac{\partial x}{\partial s}\right)_t$ in terms of the partial derivatives s_x, s_y, t_x, t_y.

$x = x(s,t)$

$$\left(\frac{\partial x}{\partial s}\right)_t = \frac{\partial x}{\partial t} \cdot \frac{\partial t}{\partial s}$$

$$S_x = \frac{\partial s}{\partial x} \qquad t_x = \frac{\partial t}{\partial x} \qquad \partial t = t_x \partial x$$

$$S_y = \frac{\partial s}{\partial y} \qquad t_y = \frac{\partial t}{\partial y} \qquad \partial t = t_y \partial y$$

$$\partial s = S_x \partial x$$

$$\partial s = S_y \partial y$$

$$\frac{\partial x}{\partial s} = \frac{\partial x}{\partial t} \cdot \frac{\partial t}{\partial s}$$

$$\left(\frac{\partial x}{\partial s}\right)_t = \frac{\cancel{\partial x}^{1}}{t_x \cancel{\partial x}} \cdot \frac{t_y \cancel{\partial y}^{1}}{S_y \cancel{\partial y}} = \boxed{\frac{t_y}{t_x S_y}}$$

X

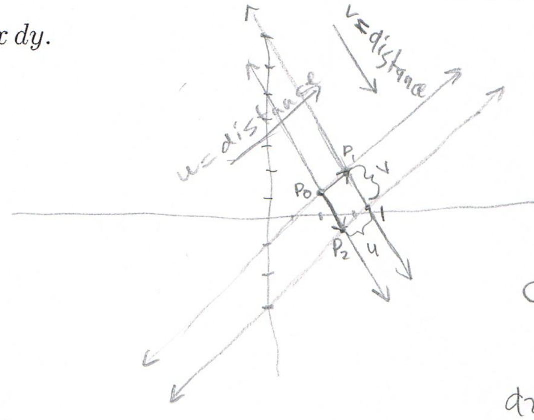
DID NOT APPLY
TOTAL DERIVATIVES

$\boxed{0}$

9) Let R be the parallelogram in $\mathbb{R}^2$ bounded by the lines

$$y = x - 1, \qquad y = x - 3, \qquad y = 5 - 2x, \qquad y = 7 - 2x.$$

Evaluate $\displaystyle\iint_R \frac{2x+y}{x-y}\, dx\, dy.$



Area $= |\overrightarrow{P_0P_1} \times \overrightarrow{P_0P_2}|$

Let $u = x+y$ ✓ $\quad x = u - y$

$v = -2x + y \quad v = -2(u-y)$

$v = -2u - 2y$

$v = -2$

$dx\,dy = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix} du\,dv$

$\displaystyle\iint dx\,dy \Rightarrow \int_0^1\int_0^1 3\,du\,dv$

$dx\,dy = \begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} du\,dv$ ✓

$1 + 2 = 3$

$\displaystyle\int_0^1\int_0^1 \frac{2(v-u)+(2u-v)}{(v-u)-(2u-v)}\, du\,dv =$

$\displaystyle\int_0^1\int_0^1 \frac{v}{2v+2u}\, du\,dv$ should be $\frac{u}{v}$

$\displaystyle\int_0^1\int_0^1 \frac{1}{2v} + \frac{v}{2u}\, du\,dv$

$\displaystyle\int_0^1 \frac{u}{2v} + \frac{-v}{2u^2}\Big|_0^1 dv =$

$\displaystyle\int_0^1 \frac{1}{2v} - \frac{1}{2}v\, dv$

$\displaystyle -\frac{1}{2v^2} - \frac{1}{4}v^2\Big|_0^1 = \boxed{\frac{1}{4}}$

ARITHMATIC ERROR
-8

$V = -2u - y$

$y = 2u - v$

$x = u - 2u + v$

$x = v - u$

$$\int_0^1\int_0^1 \frac{v}{2v+2u}\,dv\,du$$

$$\frac{1}{2}\int_0^1 \frac{1}{u}\,du$$

10) Let $\mathbf{F}$ be a vector field defined everywhere on $\mathbb{R}^3$ except the origin, pointing radially outward with magnitude $|\mathbf{F}| = 1/\rho$, where ρ is the distance to the origin. Let S_R be the sphere of radius R centered at the origin.

(a) (15 pts.) Compute the outward flux of $\mathbf{F}$ across S_R, in terms of the positive number R.

$$\iint \vec{F}\cdot\hat{n}\, dS \qquad \hat{N} = \langle 1, \varphi, \theta\rangle \cdots \hat{n} = \qquad \vec{F}$$

2x

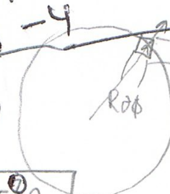
$$\hat{n}\,dS = \langle -f_x, -f_y, 1\rangle\, dx\, dy$$

$$\hat{n}\,dS =$$

FORGOT TO MULTIPLY -4

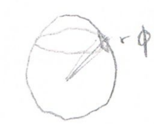
dot product $= |F||\hat{n}|\cos\theta$

$$\int_0^{2\pi}\int_0^{\pi} \frac{1}{R} r^2\, d\phi\, d\theta$$



$1/R \cdot \cos$

$1/R$



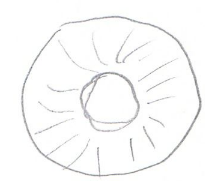
$$\int_0^{2\pi}\int_0^{\pi} R\, d\phi\, d\theta = \int_0^{2\pi} \pi R\, d\theta$$

$$\boxed{= 2\pi^2 R}$$

(b) (10 pts.) Let T be the 3D region between the spheres S_1 and S_3, i.e., the region where

$$1 \le \sqrt{x^2+y^2+z^2} \le 3.$$

What is $\iiint_T \operatorname{div}\mathbf{F}\, dV$?



DID NOT APPLY DIVERGENCE THEOREM

$$\iiint_{S_3} \operatorname{div} F\, dV - \iiint_{S_1} \operatorname{div} F\, dV$$

X $\boxed{0}$

$$\text{divergence} = \nabla\cdot\vec{F}$$

$$\left\langle \frac{x^2}{\sqrt{x^2+y^2+z^2}}, \frac{y^2}{\sqrt{x^2+y^2+z^2}}, \frac{z^2}{\sqrt{x^2+y^2+z^2}} \right\rangle$$

$$|F| = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

$$\cancel{f} =$$

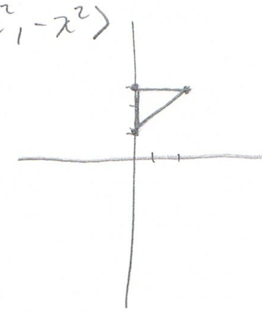
$$\hat{f} = \langle x^2, y^2, z^2\rangle$$

$\boxed{\text{UNFINISHED}}$

11) Let R be the solid triangle in $\mathbb{R}^2$ with vertices at $(0,1)$, $(2,3)$, and $(0,3)$. Let C be its boundary traversed counterclockwise. Let $\mathbf{F} = x^2(\mathbf{i} - \mathbf{j})$.

(a) (20 pts.) Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ by converting it to a double integral over the region R.

$\vec{F} = \langle x^2, -x^2 \rangle$



$$\iint_R \operatorname{curl}(F)\, dx\, dy$$

$$\int_1^2 \int_0^{y-1} -2x \, dx\, dy \quad ✓$$

$\operatorname{curl}(F) = \nabla \times F$

$\begin{vmatrix} \partial_x & \partial_y \\ \cancel{M} & \cancel{N} \end{vmatrix} \quad N_x - M_y$

$= -2x - 0 = -2x$ ✓

$$\int_1^2 -x^2 \Big|_0^{y-1} dy$$

$$\int_1^2 -(y-1)^2 \, dy$$

ARITHMATIC ERROR −4

$$\int_1^2 -y^2 + 2y - 1 \, dy$$

$$-\frac{y^3}{3} + y^2 - y \Big|_1^2 = \left(-\frac{(2)^3}{3} + (2)^2 - (2)\right) - \left(\frac{-(1)^3}{3} + (1)^2 - (1)\right) \qquad 2 - \frac{7}{3} = \boxed{-\frac{1}{3}}$$

$$-\frac{8}{3} + 4 - 2 + \left(\frac{1}{3}\right)$$

(b) (5 pts.) Is there a function $f(x, y)$ whose gradient equals $\mathbf{F}$ everywhere? (Explain your reasoning.)

No because curl $\neq 0$, the function is not conservative and therefore cannot be a gradient. ✓

12) Set up an iterated integral in *cylindrical* coordinates whose value is the moment of inertia of a solid spherical planet of radius a and constant density δ with respect to an axis through the center of the planet. (Assume that the center of the planet is at the origin, and that the axis of rotation is the z-axis.)

Do not evaluate the integral!

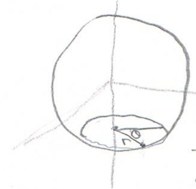
MI = sum of all mass • distance to axis of rotation

$$\boxed{\int_{-a}^{a}\int_{0}^{2\pi}\int_{0}^{\sqrt{a^2-z^2}} \delta \cdot r^2 \; r\,dr\,d\theta\,dz}$$

conversion from dx dy

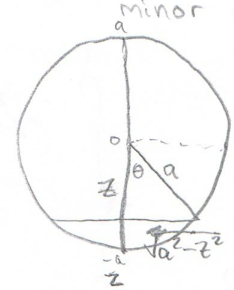
mass per unit volume x distance from axis

✓



taking slices of the planet

size of radius for minor circle



$\cos(\theta) = \frac{z}{a}$

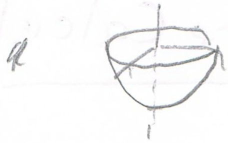
$\sin(\theta) = \frac{r}{a}$

$\cos^{-1}\left(\frac{z}{a}\right) =$

$\sin\left(\cos^{-1}\left(\frac{z}{a}\right)\right)$

$a\sin\left(\cos^{-1}\left(\frac{z}{a}\right)\right)$

13) Let S be the lower hemisphere defined by $x^2 + y^2 + z^2 = 1$ and $z \le 0$. Let $\mathbf{F} = \langle \underbrace{y + xz}_{P}, \underbrace{5 - x}_{Q}, \underbrace{2e^x}_{R} \rangle$. Compute the outward (i.e., downward) flux of $\operatorname{curl}\mathbf{F}$ across S.



compute downward flux of the curl of (F)

$$\text{Curl}(F) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ P & Q & R \end{vmatrix} = \langle R_y - Q_z, + P_z - R_x, +1$$

$$= \langle 0-0, x - 2e^x, -1-y \rangle$$

X

$\boxed{0}$

$$= \langle 0, x - 2e^x, 1-y-1 \rangle$$

ON THIS SIDE

$$\hat{n}dS = \langle f_x, f_y, -1 \rangle\, dx\,dy$$
$$= \langle 2x, 2y, -1 \rangle\, dx\,dy$$

DID NOT USE STOKE'S THEOREM

$$\iint 2xy - 4ye^x + y + 1 \; dx\,dy$$

$\Downarrow$ polar

$x = r\cos\theta$
$y = r\sin\theta$

$$\int_0^{2\pi}\int_0^1 \left(2\sin\theta\cos\theta r^2 - 4r\sin\theta e^{r\cos\theta} + r\sin\theta + 1\right) r\,dr\,d\theta$$

$\Downarrow$

$$\int_0^{2\pi}\int_0^1 r^3\sin^2\theta - 4r^2\sin\theta e^{r\cos\theta} + r^2\sin\theta + r \; dr\,d\theta$$

$$\int_0^{2\pi} \frac{r^4}{4}\sin^2\theta - \frac{4r^3}{3}\cdot\cos\theta\sin\theta\, e^{r\cos\theta} + \frac{r^3}{3}\sin\theta + r^2 \Big|_0^1 \; d\theta$$

This is the end!

$$\int_0^{2\pi} \frac{1}{2}\sin\theta\cos\theta - \frac{4}{3}\sin\theta\cos\theta\, e^{\cos\theta} + \frac{1}{3}\sin\theta + 1 \; d\theta$$