Scott Young
Exam 2012

Subject 24.241. Logic I. Sample Final Exam.
The real exam will be Wednesday, December 15, from 1:30 to 4:30, in the usual place (26-314). It's an open book exam, so you many bring your notes and any other written materials you like. Whatever materials you think you'll need will need to be printed out on paper. I apologize to the forest for her children that will be wastefully sacrificed, but the possibilities of mischief with people bringing laptops and smartphones to an exam are too ominous.
There will be a REVIEW SESSION the evening before the exam, Tuesday the 14th, at 6:00 in 32-D831. There will be pizza. Just bring questions.

1. Use the method of truth tables to determine whether the following SC sentence is tautological, inconsistent, or indeterminate: $\neg((\neg P \leftrightarrow Q) \rightarrow (((P \wedge \neg Q) \wedge R) \leftrightarrow \neg((Q \rightarrow P) \rightarrow \neg R)))$

2. Translate the following argument, using "Cxy" to translate "x is in y's class," "Ex" to translate "x is an eighth grader," "Sx" to translate "x takes Spanish," and "p" to translate "Señor Pedini." Then derive the translated conclusion from the translated premises:

   Everyone in Señor Pedini's class takes Spanish.
   Not every eighth grader takes Spanish.
   Therefore, not every eighth grader is in Señor Pedini's class.

3. Translate the following argument, taking the domain to consist of the players, and using "r" to translate "Reid," "n" to translate "Nyere," "Bxy" to translate "x is bigger than y," "Sxy" to translate "x is slower than y,"and "Lx" to translate "x is on the starting lineup." Then derive the conclusion from the premises:

   Every player bigger than Reid is slower than Reid.
   Every player slower than Reid is slower than Nyere.
   There are players on the starting lineup bigger than Reid.
   Therefore, there are players on the starting lineup slower than Nyere.

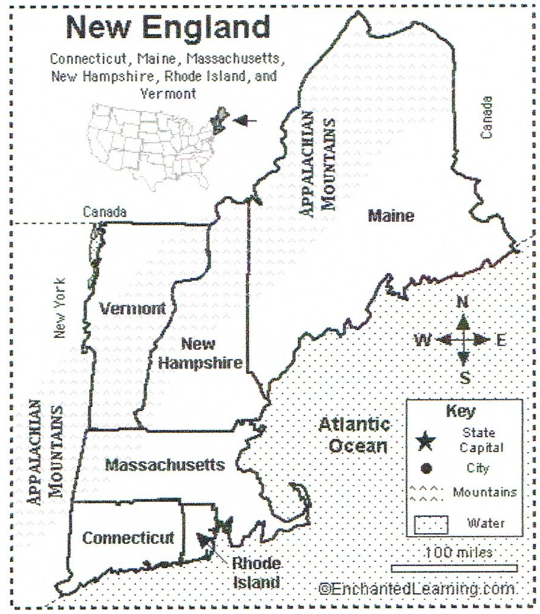
4. Symbolize the following argument, using "D" to translate "dances with," "L" to translate "loves," "j" to translate "Jasmin," and "m" to translate "Meisun"; then derive the translated conclusion from its premises:

   Everyone who dances with Jasmin loves her.
   No one loves both Jasmin and Meisun
   Therefore no one who loves Meisun dances with Jasmin.

5. Translate the following argument, using "Dx" to translate "x is a dog," "Bx" to translate "x loves to bay at the moon," "Cx" to translate "x chases cats," "s" to translate "Sola," and "t" to translate "Tarmin." Then derive the translated conclusion from the translated premises:

   Every dog other than Sola loves to bay at the moon.
   Sola is a dog who chases cats, whereas Tarmin is a dog who doesn't chase cats.
   Therefore, Tarmin loves to bay at the moon.

6. Let $\mathfrak{A}$ be an interpretation whose domain consists of the six New England states (Maine, New Hampshire, Vermont, Massachusetts, Rhode Island, and Connecticut), with $\mathfrak{A}$("*S*") = {<*x*,*y*>: *x* and *y* are in $|\mathfrak{A}|$ and *x* is to the south of *y*}, and $\mathfrak{A}$("*n*") = New Hampshire. (Here I'm thinking of all the other states as being to the south of Maine, although they're more southwest than south, really. All the states but Vermont, New Hampshire, and Maine are to the south of Vermont, and all the states but Vermont, New Hampshire, and Maine are to the south of New Hampshire. For the other states, what's to the south of what is clear from the map.) $\mathfrak{A}$("=") is, of course, equal to{<*x*,*x*>: *x* is a New England state} Let σ be the variable assignment that assigns Massachusetts to "*x*,", Vermont to "*y*," Maine to "*z*," and Rhode Island to every other variable.



a) Does σ satisfy "$(Sxn \leftrightarrow Szn)$" under $\mathfrak{A}$? Explain your answer.

b) Does σ satisfy "$(\exists y)(\exists z)(\neg y{=}z \wedge (Syx \wedge Szx))$" under $\mathfrak{A}$? Explain your answer.

7. Show that, if Γ is a nonempty set of predicate calculus sentences with the property that the conjunction of any two members of Γ is a member of Γ, then every sentence that a consequence of Γ is a consequence of an element of Γ. Keep in mind that Γ will be infinite.

8. Show that the result in problem 6 won't continue to hold if either the condition that Γ is nonempty or the condition that the conjunction of any two members of Γ is in Γ is omitted.

24.241 - Exam — Scott Young, April 13, 2012

1)

	P	Q	R	¬	((¬	P	↔	Q)	→	(((P	∧	¬	Q)	∧	R)	↔	¬	((Q	→	P)	→	¬	R)))
	T	T	T	F	F	T	F	T	T	T	F	F	T	F	T	F	T	T	T	T	F	F	T
	T	T	F	T	F	T	F	T	F	T	F	F	T	F	F	T	F	T	T	T	T	T	F
	T	F	T	F	F	T	T	F	T	T	T	T	F	T	T	T	T	F	T	T	F	F	T
	T	F	F	F	F	T	T	F	T	T	T	T	F	F	F	T	F	F	T	T	T	T	F
	F	T	T	F	T	F	T	T	T	F	F	F	T	F	T	T	F	T	F	F	T	F	T
	F	T	F	F	T	F	T	T	T	F	F	F	T	F	F	T	F	T	F	F	T	T	F
X	F	F	T	F	T	F	F	F	T	F	F	T	F	F	T	F	T	F	T	F	F	F	T
	F	F	F	T	T	F	F	F	F	F	F	T	F	F	F	T	F	F	T	F	T	T	F

indeterminate   inconsistent

2)

$$(\forall x)(Cxp \rightarrow Sx)$$
$$(\exists y)(Ey \wedge \neg Sy)$$
$$\overline{(\exists z)(Ez \wedge \neg Czp)}$$

1.	$(\forall x)(Cxp \rightarrow Sx)$	A
2.	$(\exists y)(Ey \wedge \neg Sy)$	A
3.	$Ea \wedge \neg Sa$	∃E A
4.	$Cap \rightarrow Sa$	1. ∀E
5.	Cap	¬E A
6.	Sa	4. →E
7.	$\neg Sa$	3. ∧E
8.	$\neg Cap$	(5-7) ¬E
9.	Ea	3. ∧E
10.	$Ea \wedge \neg Cap$	8-9 ∧I
11.	$(\exists z)Ez \wedge \neg Czp$	10. ∃I
12.	$(\exists z)Ez \wedge \neg Czp$	(3-11) ∃E

✓

3)

$$(\forall x)(Bxr \rightarrow Sxr)$$
$$(\forall y)(Syr \rightarrow Syn)$$
$$(\exists z)((Bzr) \wedge Lx)$$
$$\overline{(\exists w)((Swn) \wedge Lx)}$$

1.	$(\forall x)(Bxr \rightarrow Sxr)$	A
2.	$(\forall y)(Syr \rightarrow Syn)$	A
3.	$(\exists z)(Bzr)$	A
4	Bar	∃E A
5.	$Bar \rightarrow Sar$	1. ∀E
6	$Sar \rightarrow San$	2. ∀E
7	Sar	5. →E
8	San	6. →E
9	$(\exists w)Swn$	8. ∃I
10.	$(\exists w)Swn$	4-9 ∃E

4) $(\forall x)(Dxj \to Lxj)$

$\neg(\exists x)(Lxm \land Lxj)$

$\neg(\exists x)(Lxm \land Dxj)$

1	$(\forall x)(Dxj \to Lxj)$	A
2	$\neg(\exists x)(Lxm \land Lxj)$	A
3	$(\exists x)(Lxm \land Dxj)$	$\neg$I A
4	$Lam \land Daj$	$\exists$E A
5	$Daj \to Laj$	1. $\forall$E
6	Daj	4. $\land$E
7	Laj	5. $\to$E
8	Lam	4. $\land$E
	$Lam \land Laj$	7.8 $\land$I
9	$(\exists x)(Lxm \land Lxj)$	8 $\exists$I
10	$(\exists x)(Lxm \land Lxj)$	4-9 $\exists$E
11	$\neg(\exists x)(Lxm \land Lxj)$	2 R
12	$\neg(\exists x)(Lxm \land Dxj)$	3-11 $\neg$I

5) $(\forall x)(((\neg x = s) \land (Dx)) \to Bx$

$(Cs \land \neg Ct) \land (Ds \land Dt)$

Bt

1.	$(\forall x)(((\neg x = s) \land (Dx)) \to Bx)$	A
2.	$(Cs \land \neg Ct) \land (Ds \land Dt)$	A
3.	$((\neg t = s) \land (Dt)) \to Bt)$	1. $\forall$E
4.	$Cs \land \neg Ct$	2. $\land$E
5.	$Ds \land Dt$	2. $\land$E
6.	Cs	4 $\land$E
7.	$\neg Ct$	4 $\land$E
8.	Ds	5 $\land$E
9.	Dt	5 $\land$E
10.	$t = s$	$\neg$I A
11.	Cs	6. R
12.	$\neg Cs$	7,10 =
13.	$\neg t = s$	10-12 $\neg$
14.	$(\neg t = s) \land (Dt)$	13,9 $\land$
15.	Bt	3. $\to$

6) a) Does σ satisfy $(Sxn \leftrightarrow Szn)$?

Mass. is south of New Hampshire iff Maine is to the south of New H

No, it does not satisfy. Mass is south of NH, so this results in Sxn being true on assignment σ and interpretation $\mathfrak{U}$. However Maine is not south of NH, so Szn is false on ass. σ and int. $\mathfrak{U}$. T leads to the biconditional $Sxn \leftrightarrow Szn$ being false.

b) $(\neg y = z \land (Syx \land Szx))$

False False —

True False

No. It does not satisfy. Syx is false under this interpretation and variable assignment, therefore the two conjunctions are also false.

7) Let P be any sentence of predicate calculus such tha
$\Gamma \vDash P$, that is, P is entailed by (or is a consequent of) Γ
Let Γ^* be some subset of Γ such that $\Gamma^* \subseteq \Gamma$ and
$\Gamma^* \vDash P$. By result 11.4.1 completeness metatheorem (
if $\Gamma \vDash P$ then $\Gamma \vdash P$, therefore $\Gamma^* \vdash P$.
Now let Q be the iterated conjunction of all senten
in Γ^*, being $(((\cdots(\Gamma_1^* \wedge \Gamma_2^*) \wedge \Gamma_3^*)\cdots \wedge \Gamma_n^*)$. We can prove
Q is a member of Γ by mathematical induction:

Base case : Γ_1^* is a member of Γ

Induction Hypothesis: Assume Γ_i^* be a member of Γ with i conjunctions (at most). By the statec properties of Γ, $(\Gamma_i^* \wedge \Gamma_{i+1}^*)$ must als be a member of Γ

Therefore Q is a member of Γ

We will now prove that $Q \vdash P$. Begin the derivat with Q as the only open assumption. From there we apply the conjunctive elimination rule <u>n</u> times to conclude all members of Γ^*. As we now have all members of Γ^* on lines $2-n+1$, we can conclud P since $\Gamma^* \vdash P$, by our earlier result.

Following metatheorem 11.3.1 (soundness of PD) we can conclude $Q \vDash P$. Since Q is a member of Γ and P is any sentence entailed by some subset of Γ, $(\Gamma^* \subseteq \Gamma)$, we can conclude any sentence th is a consequent of Γ is a consequent of one elem of Γ.

Q.E.D.

8) a) If Γ is empty, then P must be a theorem. ($\{\emptyset\} \vDash P$)
However since Γ has no members then no such member, Q, can entail P. Therefore there are sentences which are a consequent of Γ, but of no member (since none exist) in Γ. Q.E.D.

b) If the conjunction rule is omitted, there will be sentences only entailed by some subset of Γ larger than one element, or $|\Gamma^*| > 1$. As such there is no single element, Q, which entails P.
For example, $\Gamma = \{A, B\}$, $P: \{A \wedge B\}$

$$A \nvDash A \wedge B$$
$$B \nvDash A \wedge B$$
$$\{A, B\} \vDash A \wedge B$$ (By $\wedge$I Rule and the soundness and completeness metatheorems of PD)

GRADING

1.	0/10
2.	10/10
3.	5/10
4.	10/10
5.	10/10
6.	5/10
7.	10/10
8.	10/10
	75%

This exam used slightly different solutions however I believe my answers are valid except as otherwise indicated. In question 3, I subtracted half points for a mistranslation, however in all other cases I believe my answers are quantificationally equivalent to those shown.