LOGIC II - 24.242

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HOMEWORK # 1

1. Write down a bounded formula whose extension is the set of triples $\langle x, y, z \rangle$ such that x, y and z are positive integers and z is a common divisor of x and y.

We want x, y and z such that:

$$(\exists a)\, z \cdot a = x$$
$$(\exists b)\, z \cdot b = y$$

replace this w/ non-negativity constraint

$$\cancel{(\forall x < \mathcal{N})(\forall y < \mathcal{N})(\forall z < \mathcal{N})}((\exists a < sx)(z \cdot a) = x) \wedge (\exists b < sy) \ldots ((z \cdot b) = y))$$

use $\mathcal{N}$ because question required a bounded formula.

Provided answer:

$$(((0 < x \wedge 0 < y) \wedge 0 < z) \wedge \ldots$$
$$\ldots ((\exists u < sx)(u \cdot z) = x) \wedge (\exists v < sv)(v \cdot z) = y))$$

*Note – variables can be free in the bound formula!

2. Define, for F, a finite set of the natural numbers, Code(F) to be $\sum_{x \in F} 2^x$, so that F is the set of places in the binary decimal expansion where 1s appear.

Give the Arabic numeral for Code($\{2, 4, 6, 8\}$)

$$\text{Code}(\{2,4,6,8\}) = 2^2 + 2^4 + 2^6 + 2^8$$
$$4 + 16 + 64 + 256$$
$$= 340$$

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HOMEWORK #2

1. Write a program that calculates $(x + y)$ [using the register program model provided]

1. If the number in register 2 is zero, go to step 5.
2. Subtract 1 from register 2, unless that number is already zero.
3. Add 1 to the number in register 1.
4. Go to instruction 1.
5. STOP.

2. Show that a set is Δ iff its characteristic function, χ_s, is Σ.

If a set is Δ, this means that we can effectively enumerate all members and non-members. This implies its characteristic function χ_s recursively enumerable because we can output "1" if [n] is an element of s and "0" if it is not an element, therefore χ_s is Σ

This proves S is $\Delta \Rightarrow \chi_s$ is Σ

If a function χ_s is Σ and a characteristic function, it must return "1" or "0" for all [n]. This produces the bounded formula $({}_s0) = \chi_s([n])$. Since all bounded formulas extend to Δ this proves

χ_s is $\Sigma \Rightarrow S$ is Δ