

HOMWORK # 1

1. Write down a bounded formula whose extension is the set of triples $\langle x, y, z \rangle$ such that x, y and z are positive integers and z is a common divisor of x and y .

We want x, y and z such that:

$$(\exists a) z \cdot a = x \quad \checkmark$$

$$(\exists b) z \cdot b = y \quad \checkmark$$

replace this w/ non-negativity constraint

~~$$(\forall x < \tau)(\forall y < \tau)(\forall z < \tau)((\exists a < \tau)(z \cdot a = x) \wedge (\exists b < \tau)(z \cdot b = y))$$~~

↑ Use τ because question required a bounded formula. ... $((z \cdot b) = y)$

Provided answer:

$$((0 < x \wedge 0 < y) \wedge 0 < z) \wedge \dots$$

$$\dots ((\exists u < sx)(u \cdot z) = x) \wedge (\exists v < sv)(v \cdot z) = y)$$

* Note - variables can be free in the bound formula!

2. Define, for F , a finite set of the natural numbers, $\text{Code}(F)$ to be $\sum_{x \in F} 2^x$, so that F is the set of places in $x \in F$ the binary decimal expansion where 1s appear.

Give the Arabic numeral for $\text{Code}(\{2, 4, 6, 8\})$

$$\begin{aligned} \text{Code}(\{2, 4, 6, 8\}) &= 2^2 + 2^4 + 2^6 + 2^8 \\ &= 4 + 16 + 64 + 256 \\ &= 340 \quad \checkmark \end{aligned}$$

[Faint, illegible handwriting]

[Faint, illegible handwriting]

[Faint, illegible handwriting]

[Faint, illegible handwriting]

[Faint, illegible handwriting]

LOGIC II + 24.242

HOMWORK #2

1. Write a program that calculates $(x + y)$ [using the register program model provided]

1. If the number in register 2 is zero, go to step 5.

2. Subtract 1 from register 2, unless that number is already zero.

3. Add 1 to the number in register 1.

4. Go to instruction 1.

5. STOP.

2. Show that a set is Δ iff its characteristic function, χ_s , is Σ .

If a set is Δ , this means that we can effectively enumerate all members and non-members. This implies its characteristic function χ_s recursively enumerable because we can output "1" if $[n]$ is an element of s and "0" if it is not an element, therefore χ_s is Σ .

This proves S is $\Delta \Rightarrow \chi_s$ is Σ

If a function χ_s is Σ and a characteristic function, it must return "1" or "0" for all $[n]$. This produces the bounded formula $(s0) = \chi_s([n])$. Since all bounded formulas extend to Δ this proves

χ_s is $\Sigma \Rightarrow S$ is Δ