

Massachusetts Institute of Technology  
Department of Electrical Engineering and Computer Science

6.002 – Electronic Circuits  
Fall 2000

Final Exam

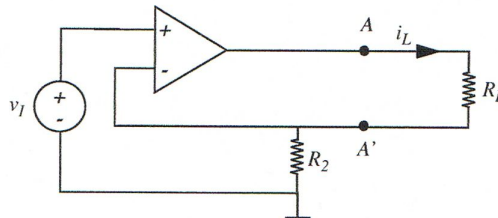
- Please write your name in the space provided below, and circle the name of your recitation instructor and the time of your recitation.
- Please verify that there are 19 pages in your exam.
- To the extent possible, do all of your work on the pages contained within this exam. In particular, try to do your work for each question within the boundaries of the question, or on the back side of the page preceding the question.
- You may use three double-sided pages of notes while taking this exam.
- Final grades in 6.002 will not be given out by phone or by e-mail. Rather, they should be available through WEBSIS by December 22. You may review and take back your final exam at the beginning of IAP in January 2001 from Room NE43-624.
- Good luck!

Problem	Score
1	5/15
2	13/15
3	11/15
4	2/15
5	0/10
6	13/15
7	9/15
Total	53.0

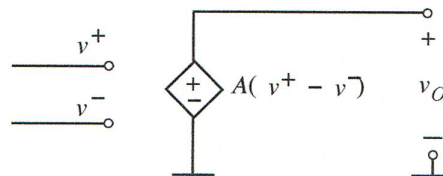
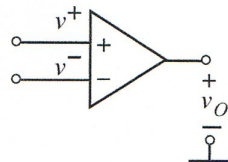
Name: SCOTT YOUNG

Problem 1 - 15 Points

The Op-Amp circuit shown below is very similar to the standard non-inverting Op Amp except that  $R_L$  is some external resistor, and we are interested in showing that the current through  $R_L$  is nearly constant, regardless of the value of  $R_L$ , that is, the circuit acts like a *current source* for driving  $R_L$ .



(A) Using the Op-Amp model shown below, derive an expression for  $i_L$  in terms of  $v_I$ ,  $A$ ,  $R_2$  and  $R_L$ . Show that this expression for  $i_L$  becomes independent of  $R_L$  as  $A$  approaches infinity.



Output voltage of Op-Amp is  $A(v^+ - v^-)$

Op-Amp  
system function:

$$\frac{A}{1 + RA}$$

$\lim_{A \rightarrow \infty} \frac{(v^+ - v^-)}{\frac{R_L}{1 + RA}} = v^+ - v^-$   
independent of  $R_L$

$$v^+ = v_I$$

$$i_L = \frac{v_{out} - v^-}{R_L}$$

$$\frac{A(v^+ - v^-) - v^-}{R_L} = i_L$$

$$\frac{Av^+ - (A+1)v^-}{R_L} = i_L$$

$$v_O = A(v^+ - v^-)$$

$$i_L = \frac{v_O}{R_L} = \frac{A(v^+ - v^-)}{R_L} \frac{v^-}{R_L} = A(v^+ - v^-)$$

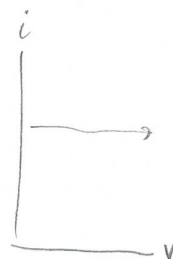
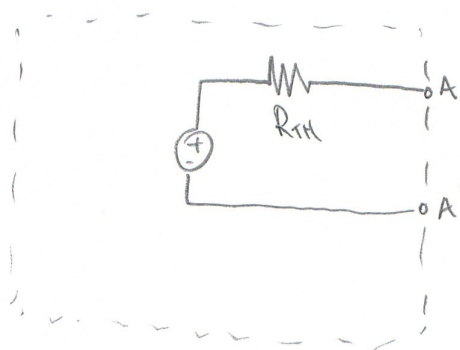
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$$\frac{(v^+ - v^-)}{\frac{R_L}{1 + RA}} - \frac{v^-}{\frac{R_L}{1 + RA}} = v^+ - v^-$$

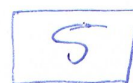


(B) To verify the “current source” action more directly, use the Op-Amp model from Part A to find the Thévenin equivalent resistance looking to the left of terminals AA’.

$$R_{TH} = \infty$$

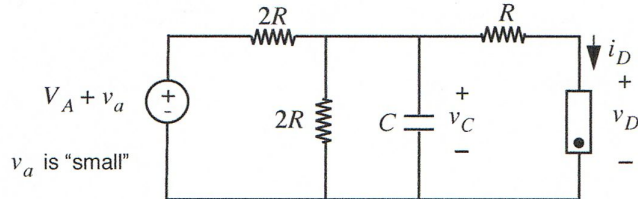


$$R_{TH} = \frac{\Delta V}{\Delta i} = \infty$$



## Problem 2 - 15 Points

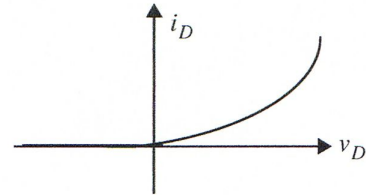
This problem involves the circuit shown below. You are given that  $R = 1\Omega$ ,  $C = 1\mu F$ , and  $K = \frac{1}{2} \left( \frac{\text{Amps}}{\text{Volt}^2} \right)$ .



Characteristics of nonlinear device:

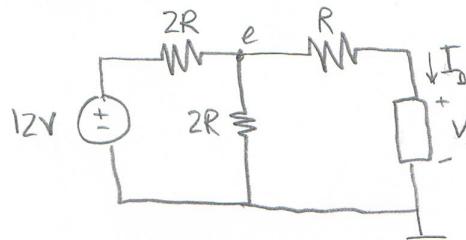
$$i_D = 0 \text{ when } v_D \leq 0$$

$$i_D = K v_D^2 \text{ when } v_D > 0$$



(A) Find the operating point voltage  $V_D$  and the operating point current  $I_D$  in the circuit shown above. Assume for this part that  $V_A = 12V$ .

For this point, we can ignore the capacitor



$$V_D = 3V$$

$$I_D = 4.5A$$

WRONG ANSWER, DUE TO SIGN SWAP

$$V_{2R}: \frac{12V - e}{2} + \frac{V_D - e}{2} + \frac{0 - e}{2} = 0$$

$$V_D: e - V_D = I_D$$

$$6 - \frac{e}{2} + V_D - e - \frac{e}{2} = 0$$

$$6 + V_D = 2e$$

$$e = 3 + \frac{V_D}{2}$$

$$0 = (V_D + 3) \left( \frac{2}{R} \right)$$

$$V_D = 3$$

$$3 + \frac{V_D}{2} = \frac{1}{2} V_D^2$$

$$0 = V_D^2 - V_D - 6$$

should be

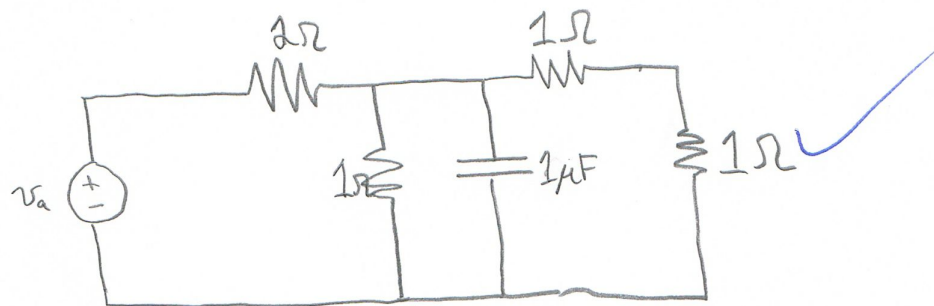
(B) Now assume that  $V_A$  is changed such that a new operating point  $I_{D0}$  and  $V_{D0}$  is obtained, where  $V_{D0} = 1V$ . (Note that this operating point may differ from your answer in Part A.)

Draw the small-signal circuit, substituting numeric values where possible. Label the small-signal capacitor voltage  $v_c$  and the small-signal device voltage  $v_d$ . (Recall, the small signal model for a capacitor is the capacitor itself.)

At  $V_{D0} = 1V$ , linear approximation = 1<sup>st</sup> derivative of  $I_D, \frac{I_D}{V}$

$$\frac{\partial I_D}{\partial V_D} \bigg|_{V_D=1} = \frac{I_D}{V_D} \bigg|_{V_D=1} = 1S = (1\Omega)^{-1}$$

= conductance



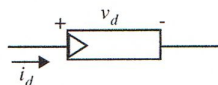


**Problem 3 – 15 Points**

A nonlinear device with voltage  $v_d$  and current  $i_d$  has a characteristic given by

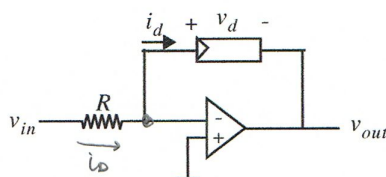
$$i_d = I e^{\alpha v_d}$$

where  $v_d > 0$  and  $i_d > 0$ .



For all parts of this problem assume that the operational amplifiers are ideal, that they are operating in the active region and that the nonlinear device is operating with positive values of  $v_d$  and  $i_d$ .

(A) For the nonlinear element connected as shown below, find an expression relating  $v_{out}$  to  $v_{in}$ .



$$v^- = v^+ = 0$$

$$i_o = \frac{v_{in}}{R}$$

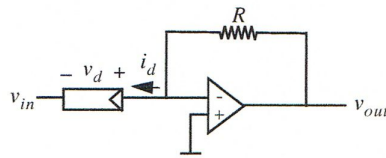
$$\frac{v_{in}}{R} = I e^{\alpha (-v_{out})}$$

$$\ln\left(\frac{v_{in}}{RI}\right) = \ln(e^{-\alpha v_{out}})$$

$$\ln\left(\frac{v_{in}}{RI}\right) = -\alpha v_{out}$$

$$v_{out} = -\frac{\ln\left(\frac{v_{in}}{RI}\right)}{\alpha}$$

(B) For the nonlinear element connected as shown below, find an expression relating  $v_{out}$  to  $v_{in}$ .



$$v^- = v^+ = 0$$

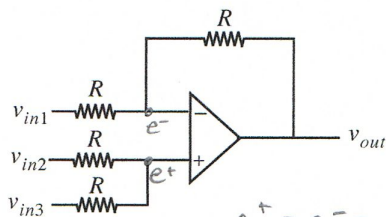
$$i_d = \frac{v_{out}}{R}$$

-2 ALGEBRA  
ERROR

$$\frac{v_{out}}{R} = I e^{-\alpha v_{in}}$$

$$v_{out} = \frac{IR}{\alpha} e^{-\alpha v_{in}}$$

(C) For the operational amplifier circuit shown below, find an expression relating  $v_{out}$  to  $v_{in1}$ ,  $v_{in2}$ , and  $v_{in3}$ .



$$e^+ = e^- = e$$

$$e^-: \frac{v_{in1} - e}{R} + \frac{v_{out} - e}{R} = 0$$

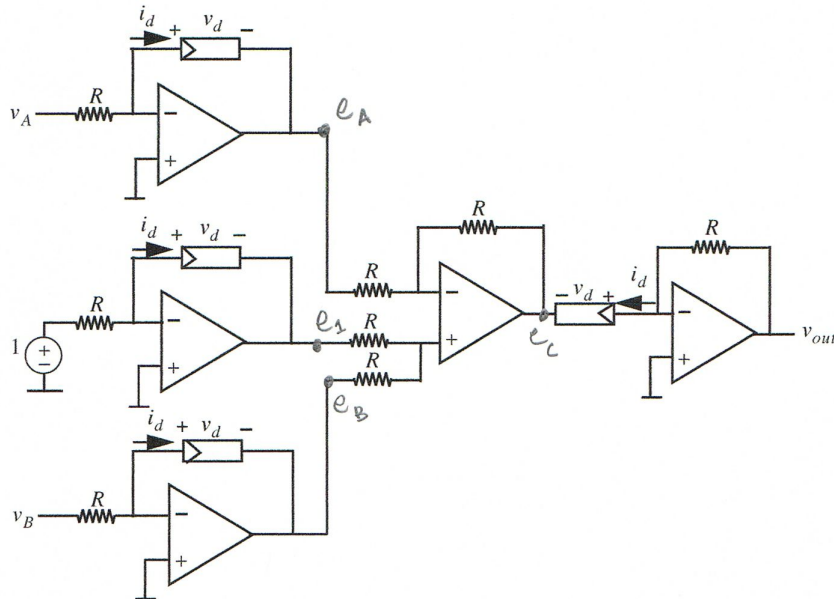
$$e^+: \frac{v_{in2} - e}{R} + \frac{v_{in3} - e}{R} = 0$$

$$\frac{v_{in1}}{R} + \frac{v_{out}}{R} = \frac{e}{R}$$

$$v_{in2} + v_{in3} = e$$

$$v_{out} = v_{in2} + v_{in3} - v_{in1}$$

(D) Find an expression for  $v_{out}$  as a function of  $v_a$  and  $v_b$  for the circuit shown below. Hint: Use the results of the previous parts of this problem.



$$e_A = \frac{-\ln\left(\frac{v_A}{RI}\right)}{\alpha} \quad e_1 = \frac{-\ln\left(\frac{1}{RI}\right)}{\alpha} \quad e_B = \frac{-\ln\left(\frac{v_B}{RI}\right)}{\alpha}$$

$$e_c = e_1 + e_B - e_A$$

$$v_{out} = \frac{I}{R} e^{-\alpha e_c}$$

$$v_{out} = \frac{I}{R} e^{-\alpha \left( \frac{-\ln\left(\frac{1}{RI}\right)}{\alpha} - \frac{-\ln\left(\frac{v_B}{RI}\right)}{\alpha} + \frac{\ln\left(\frac{v_A}{RI}\right)}{\alpha} \right)} = \frac{I}{R} \left( -\frac{1}{RI} - \frac{v_B}{RI} + \frac{v_A}{RI} \right)$$

CARRY FWD

$$v_{out} = \frac{1}{R^2} - \frac{v_B}{R^2} + \frac{v_A}{R^2}$$

$$v_{out} = v_B / v_A$$



#### Problem 4 – 15 Points

Anna Logue, a UROP student working for Professor S., has just found a rather strange piece of circuitry in the trash just outside the office of Prof. A. The circuit has three exposed terminals and it has exactly one resistor, one inductor, and one capacitor, but she cannot see how they are connected, because the connections are protected by epoxy. She can read the capacitor label: it says that the capacitor has a capacitance of  $1\mu F$  (*Observation O1*). However, the other labels are not readable.

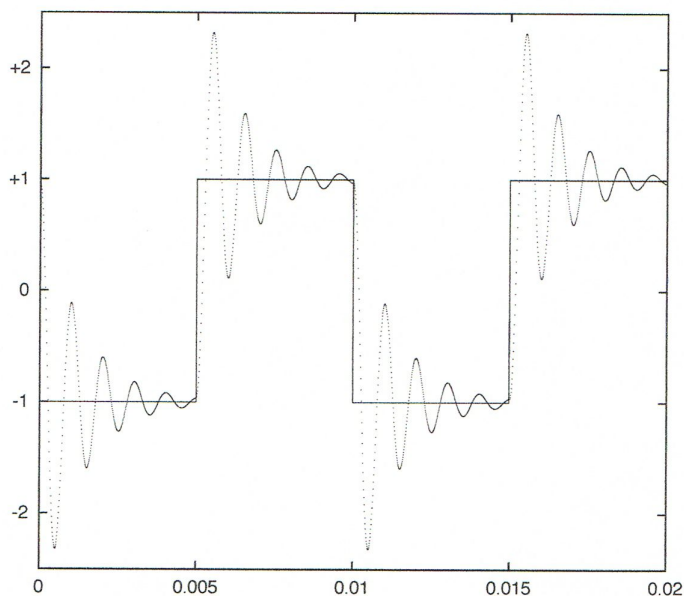
Being a naturally curious MIT student, Anna takes the circuit to the 6.002 lab to see what can be determined about this strange device by experiment. Anna labels the terminals with the letters “ $x$ ”, “ $y$ ”, and “ $z$ ” with a marker pen. She then takes several measurements. First, Anna measures the resistances between the terminals of the mystery circuit with an Ohmmeter, and makes the following observations:

$x - y$ : infinity (*Observation O2*)

$y - z$ : infinity (*Observation O3*)

$z - x$ :  $40\ \Omega$  (*Observation O4*)

Next, Anna uses a signal generator, which may be modeled as a voltage source, to apply a 1 Volt, 100 Hz, square-wave signal from the  $x$  terminal to the  $y$  terminal. She uses her oscilloscope to display the voltage from  $z$  to  $y$  superimposed on the square wave, at the same scale as the square wave, as shown in the figure below (*Observation O5*). As you can see, she observes that the waveform from  $z$  to  $y$  follows the square wave, but there is ringing with a cycle time of about 1 ms.

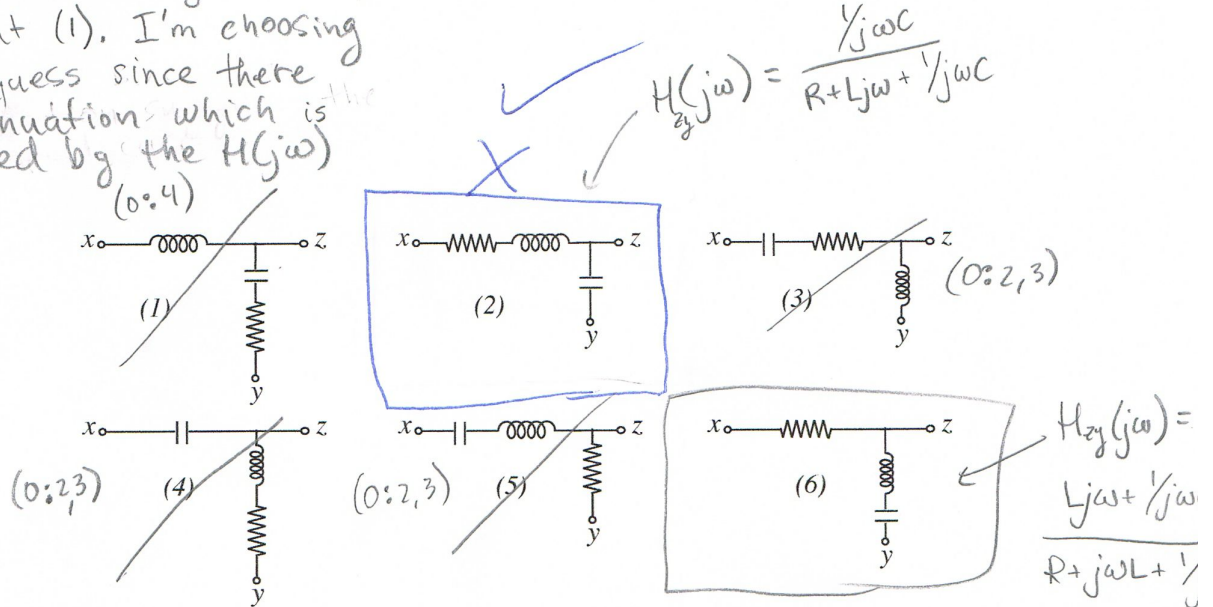


(A) Circle the circuit below which best reflects the contents of the mysterious circuit. Assume that the inductor and capacitor are ideal. Justify your choice with a two or three sentence explanation.

Assuming Ohmmeter operates at a DC bias point,  $H(j\omega)$  must be 0 between  $x$  &  $y$ . This suggests there is a capacitor between  $x$  &  $y$ . The resistance figure (0.4) also rules out (1). I'm choosing (6) as a guess since there is no attenuation which is suggested by the  $H(j\omega)$  (0.4)

- 1)  $C = 1 \mu F$
- 2)  $Z, x \leftrightarrow y = \infty$
- 3)  $Z_{yz} = \infty$
- 4)  $Z_{zx} = 40 \Omega$
- 5) Underdamped w/  $T \approx 1 \text{ ms}$

2



(B) What is the inductance (approximately) of the inductor that is in the circuit? What is the resistance of the resistor?

Assuming #2 is system:

$$1 \text{ ms} = \tau = 2R\sqrt{\frac{L}{C}}$$

$$0.001 = 2R\sqrt{\frac{L}{1 \cdot 10^{-6}}}$$

$$1 \cdot 10^{-6} = 2R^2 \frac{L}{1 \cdot 10^{-6}}$$

$$1 = 2R^2 L$$

$$L = \frac{1}{2R^2}$$

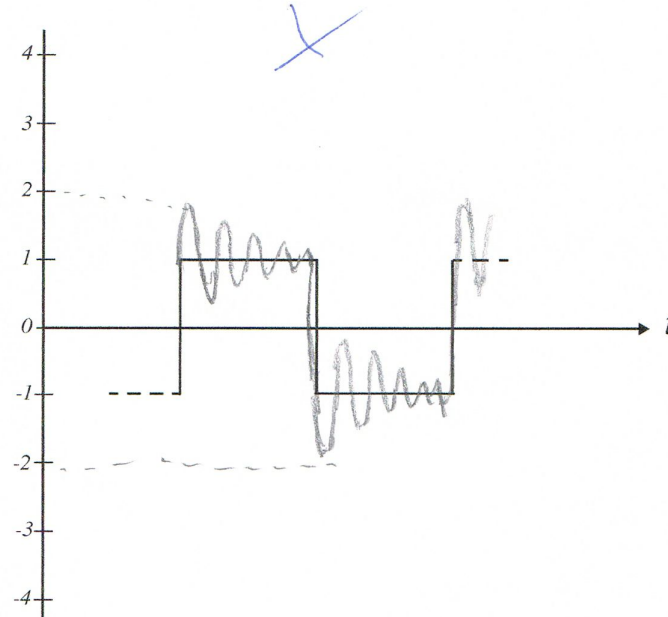
$$R = \sqrt{\frac{1}{2L}}$$

$$H_{zy}(j\omega) = \frac{Lj\omega + 1/j\omega}{R + j\omega L + 1/j\omega} = \frac{-LC\omega^2 + 1}{RCj\omega + 1 - LC\omega^2}$$

$$|H(100)| = 1 =$$

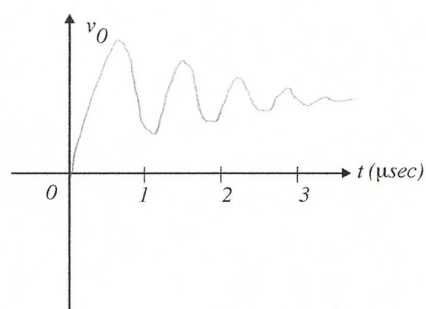
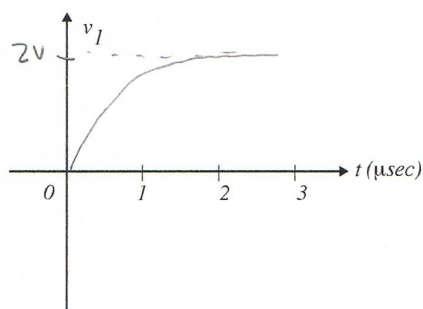
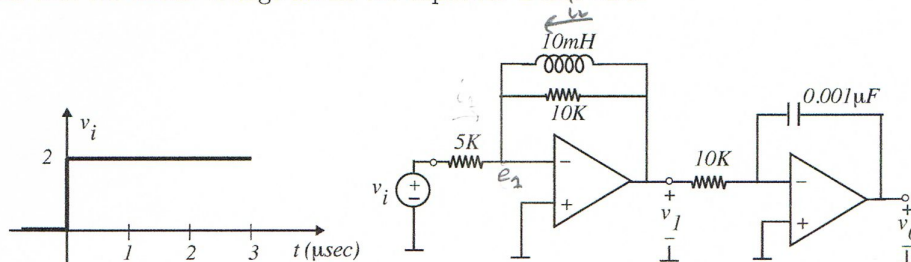
(C) Assume that Anna uses the signal generator to apply the same 1 Volt, 100 Hz, square-wave signal from the  $x$  terminal to the  $y$  terminal. For the circuit you chose in Part A, sketch the **form** of the time-domain graph of the voltage from the terminal labeled  $x$  to the terminal labeled  $z$ . Clearly indicate the values just before and just after each transition of the input.

*Basically the same as the graph on page 10.*



**Problem 5 – 10 Points**

For the network shown below, determine an expression for the indicated variables for  $t > 0$  and provide clearly labeled sketches of your result. Assume that the Op-Amps are ideal and that they are operating in the active region. Further assume that the initial current through the inductor is zero, and that the initial voltage across the capacitor is also zero.



$$v_l = \text{_____} \quad t > 0$$

$$v_o = \text{_____} \quad t > 0$$

$$e_1: \frac{v_i}{5K} + \frac{v_1}{10K} + i_L = 0$$

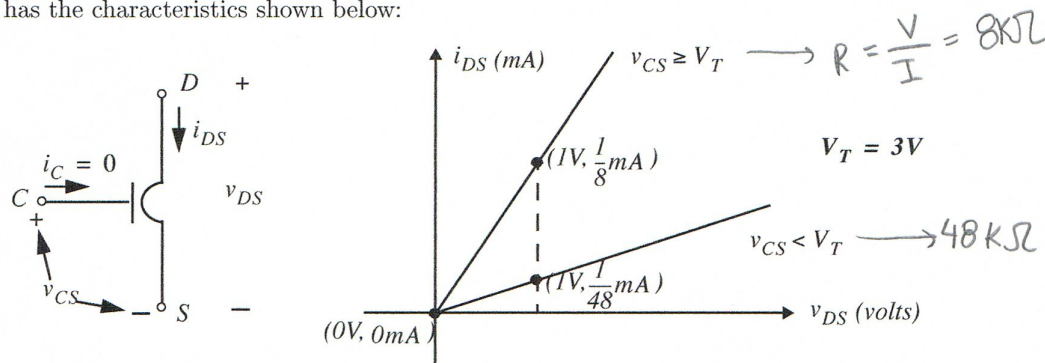
$$\frac{dv_i}{dt} \cdot \frac{1}{5K} + \frac{v_1}{10K} + L \frac{dv_1}{dt} = 0$$

X



### Problem 6 – 15 Points

Looking to recover from a disastrous product introduction, the marketing team of Yikes Inc. has changed the company name to Yehaa Inc. Furthermore, they hire several MIT undergraduates with a 6.002 background in the hopes they will turn the company around. One of the engineers discovers a new three-terminal device which can be manufactured at a lower cost than the MOSFET. This device has the characteristics shown below:

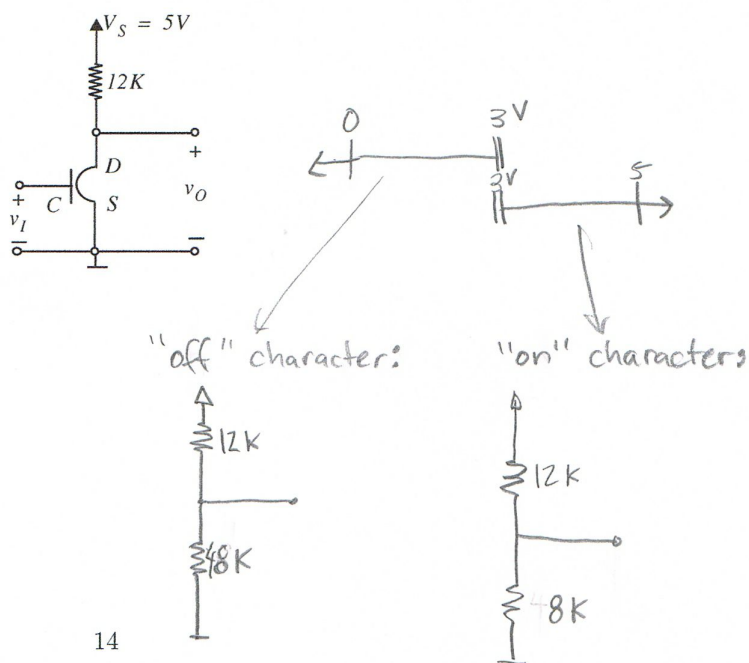


As depicted in the figure, when  $v_{CS} \geq V_T$ , the  $v_{DS}$  versus  $i_{DS}$  relationship is a straight line passing through the points  $(0V, 0mA)$  and  $(1V, \frac{1}{8}mA)$ .

When  $v_{CS} < V_T$ , the  $v_{DS}$  versus  $i_{DS}$  relationship is a straight line passing through the points  $(0V, 0mA)$  and  $(1V, \frac{1}{48}mA)$ .

The current into node C of the device is always zero. In a rare display of insight, the marketing group names this device the LOSFET.

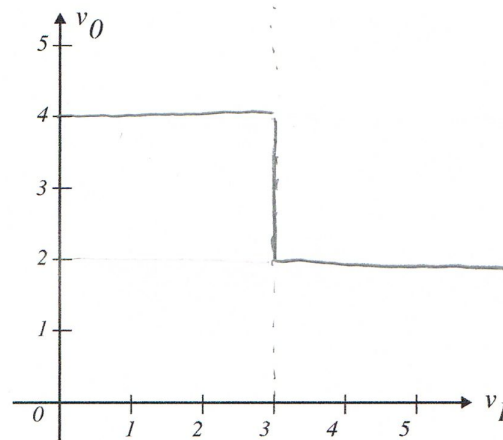
Based on their experience with MOSFETs, the Yehaa Inc. engineers use the LOSFET to build the inverter circuit shown below.





(A) Sketch  $v_I$  versus  $v_O$  for the inverter.

$$\begin{aligned} \text{OFF } (v_{in} < 3V) \\ v_{out} &= v_{th} = 5 \cdot \frac{48K}{12K + 48K} = 4V \\ \text{ON } (v_{out} > 3V) \\ v_{out} &= 5 \cdot \frac{8}{20} = 2V \end{aligned}$$



(B) Yehaa's sales team discovers that networking equipment company Disco Inc. buys huge quantities of MOSFET inverters. Disco's systems operate under a static discipline with the following voltage thresholds:

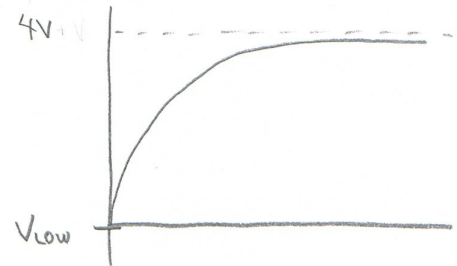
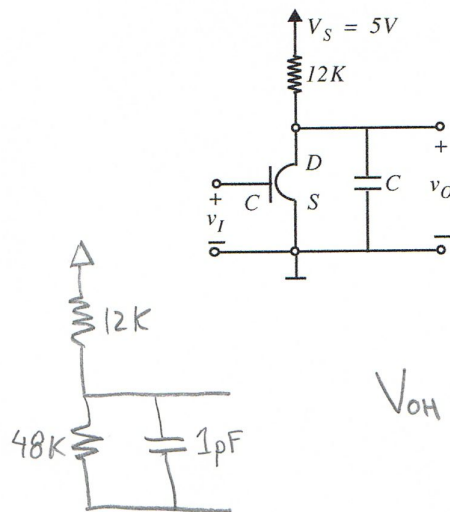
$$V_{IL} = 2.5V, V_{IH} = 3.5V, V_{OL} = 2.2V, \text{ and } V_{OH} = 3.9V.$$

Determine whether Yehaa's inverters satisfy this static discipline. Justify your answer.

Yes, they do,  
given

$$\begin{aligned} V_{IL} = 2.5V & \rightarrow \text{out} = 4V (> 3.9V) \\ V_{IH} = 3.5 & \rightarrow \text{out} = 2V (< 2.2V) \end{aligned}$$

(C) In a given situation, a Yehaa inverter drives a capacitive load with capacitance  $C = 1\text{pF}$  as shown in the figure below. Assume that the input to the inverter is at  $5\text{V}$  and that the output is at some voltage  $V_{\text{LOW}}$ . Suppose that the input switches instantaneously to  $0\text{V}$ . Find the time taken by the output of the inverter to rise to  $V_{\text{OH}}$  following the  $5\text{V}$  to  $0\text{V}$  transition at the input. Express your answer in terms of  $V_{\text{LOW}}$  and  $V_{\text{OH}}$ .



$$V_{\text{OH}} = (4 - V_{\text{LOW}})(1 - e^{-t/\tau})$$

time to rise given by this

$$R = 48\text{k} \parallel 12\text{k} = 9600\Omega$$

$$\tau = RC = 9600 \cdot 1 \cdot 10^{-12} = 9.6 \cdot 10^{-9}$$

$$\begin{matrix} \text{m} & \mu & \text{n} & \text{p} \\ 10^{-3} & 10^{-6} & 10^{-9} & 10^{-12} \end{matrix}$$

$$V_{\text{OH}} = (4 - V_{\text{LOW}})(1 - e^{-t/9.6 \cdot 10^{-9}})$$

$$1 - \frac{V_{\text{OH}}}{4 - V_{\text{LOW}}} = e^{-t/9.6 \cdot 10^{-9}}$$

$$\ln\left(1 - \frac{V_{\text{OH}}}{4 - V_{\text{LOW}}}\right) = -t/9.6 \cdot 10^{-9}$$

$$t = -9.6 \cdot 10^{-9} \ln\left(1 - \frac{V_{\text{OH}}}{4 - V_{\text{LOW}}}\right)$$

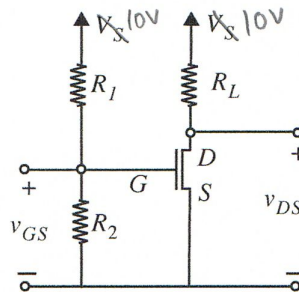
[2 ALGEBRA ERROR]

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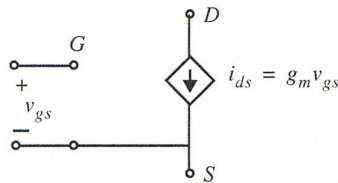
**Problem 7 - 15 Points**

Consider the amplifier circuit shown below. Assume that the MOSFET operates in saturation so that

$$i_{DS} = \frac{K}{2}(v_{GS} - V_T)^2$$



(A) A small-signal model for the MOSFET in the amplifier circuit is shown below. Given that  $V_S = 10V$ ,  $V_T = 1V$ , and  $K = 1mA/V^2$ , determine a constraint on the values of  $R_1$  and  $R_2$  such that  $g_m = 4mA/V$ .



$$g = \frac{i}{v} = \frac{1}{4} (v_{GS} - V_T)^{-1/2}$$

$$4 = (v_{GS} - V_T) = v_{GS} - 1$$

$$10 \cdot \frac{R_2}{R_1 + R_2} = 35 \quad \text{ALGEBRA ERROR} \quad \sim 2$$

$$10R_2 = 3R_1 + 3R_2$$

$$\boxed{\text{constraint: } \frac{R_1}{R_2} = \frac{10}{3}}$$

also

$$v_{GS} > V_T$$

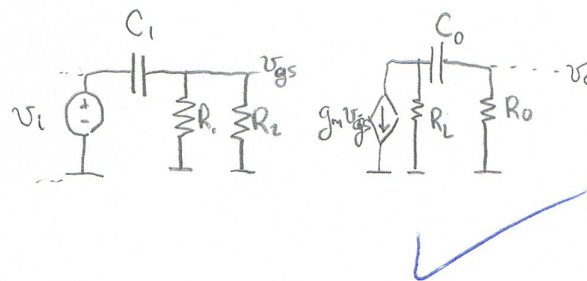
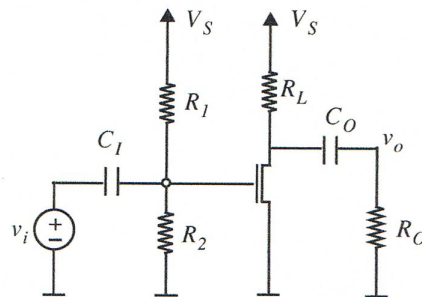
$$10 \cdot \frac{R_2}{R_1 + R_2} > 1$$

$$10R_2 > R_1 + R_2$$

$$R_2 > \frac{1}{9} R_1$$

unnecessary

(B) Draw the small-signal circuit model for the circuit shown below. Assume that the MOSFET operates in its saturation region and is biased such that the value of  $g_m$  in its small-signal circuit model is  $g_{m0}$ . Clearly label the small-signal voltages  $v_i$  and  $v_o$ .





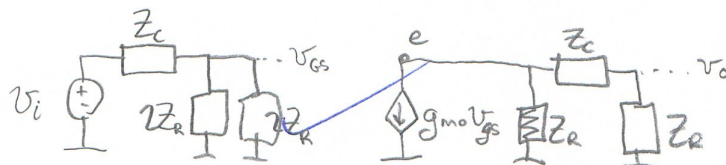
(C) Now assume that the amplifier from Part B is driven by a small-amplitude input of the form  $v_i = V_i e^{j\omega t}$ . Further assume that

$$R_L = R_O = R$$

$$R_1 = R_2 = 2R$$

$$C_I = C_O = C$$

Determine the complex small-signal gain of the amplifier  $H(j\omega) = \frac{V_o}{V_i}$  in terms of  $C, R, g_{m0}$ , and  $\omega$ . What is the magnitude of the small-signal gain of the amplifier as  $\omega$  approaches infinity.



$$v_{gs} = v_i \frac{2Z_R \parallel 2Z_R}{Z_c + 2Z_R \parallel 2Z_R}$$

$$= v_i \frac{R}{\frac{1}{j\omega C} + R} = \frac{RCj\omega}{1 + RCj\omega}$$

$$\frac{1}{R} + \frac{1}{R} = \frac{R+R}{R^2} = \frac{R^2}{2R} = \frac{R}{2}$$



$$e: -g_{m0} v_{gs} + \frac{(0-e)}{R} + \frac{v_o - e}{Z_c} = 0$$

$$v_o: \frac{(e - v_o)}{Z_c} + \frac{(0 - v_o)}{R} = 0$$

$$v_o = \frac{g_{m0} \left( v_i \frac{R}{\frac{1}{j\omega C} + R} \right) j\omega C}{\frac{1}{R} + \frac{v_o}{v_o} j\omega C} - \frac{v_o}{R} = 0$$

$$v_i \left( \frac{g_{m0} R j\omega C}{1 + j\omega RC} \right) - \frac{v_o}{R} = 0$$

\* NEED TO REDUCE!

$$-g_{m0} v_{gs} - \frac{e}{R} + \frac{v_o - e}{Z_c} = 0$$

$$e \left( \frac{1}{R} + \frac{v_o}{Z_c} \right) = g_{m0} v_{gs}$$

$$g_{m0} v_i \frac{RC^2 \omega^2}{1 + RCj\omega} = \frac{v_o}{R} \left( \frac{1}{R} + \frac{v_o}{j\omega C} \right)$$

$$\frac{g_{m0} v_{gs}}{\frac{1}{R} + \frac{v_o}{Z_c}} - \frac{v_o}{R} = 0$$

$$e = \frac{g_{m0} v_{gs}}{\frac{1}{R} + \frac{v_o}{Z_c}}$$