1

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science
6.013 Electromagnetics and Applications

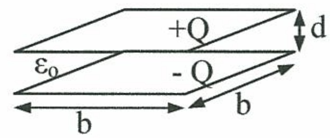
Student Name:

Final Exam   Closed book, no calculators

Please note the two pages of formulas provided at the back; the laser and acoustic expressions have been revised slightly. There are 10 problems; some are on the back sides of the sheets. For full credit, please ***simplify all expressions****, present* ***numerical answers to the extent practical*** *without a calculator or tedious computation, and place your* ***final answers within the boxes provided****. You may leave natural constants and trigonometric functions in symbolic form (π, ε_o, μ_o, η_o, h, e, sin(0.9), $\sqrt{2}$, etc.). To receive partial credit, provide all related work on the same sheet of paper and give brief explanations of your answer. Spare sheets are at the back.*

Problem 1. (25/200 points)

Two square capacitor plates in air have separation d, sides of length b, and charge ±Q as illustrated. Fringing fields can be neglected.



a) What is the capacitance C_a of this device?

$C_a = \dfrac{\varepsilon_o b^2}{\varepsilon_o d}$

$$C = \frac{Q}{V}$$

$$E_o = \frac{Q\varepsilon_o}{b^2}$$

$$V = E_o d = \frac{Q\varepsilon_o d}{b^2}$$

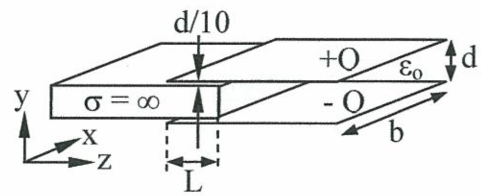
$$C = \frac{Q}{\frac{Q\varepsilon_o d}{b^2}} = \frac{b^2}{\varepsilon_o d}$$

4/8

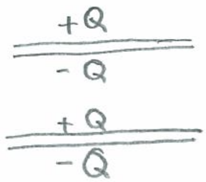
NOTE: I deducted half marks for flipping ε_o

Please turn sheet over to answer parts (b) and (c).

b) A perfectly conducting plate is introduced between the capacitor plates, leaving parallel gaps of width d/10 above and below itself. What now is the device capacitance C_b when it is fully inserted?



$C_b = \frac{5b^2}{\varepsilon_0 d}$



Since there cannot be charge within the interior a perfect conductor the result is to maintain the same charge over a smaller distance

4/8

$C = \frac{b^2}{\varepsilon_0 (\frac{1}{10}) d}$

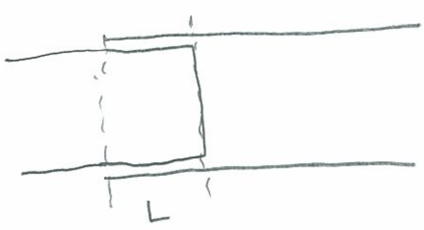
Here I deducted half marks for forgetting the ½ factor of series capacitors

c) What is the magnitude and direction of the force $\bar{f}$ on the new plate of Part (b) as a function of the insertion distance L. Please express your answer as a function of the parameters given in the figure.

$\bar{f} = -.45 \frac{Q^2 \varepsilon_0 d}{bL^2}$

Using the energy method,

$W_c = \frac{1}{2} C V^2$



$W_e = \frac{1}{2} C V^2$

X 0/9

$= \frac{1}{2} C \left(\frac{Q}{C}\right)^2$

$= \frac{\frac{1}{2} Q^2}{C}$

$F = \frac{W_e}{L}$

$\Delta W_e = \frac{\frac{1}{2} Q^2}{C'^2} - \frac{\frac{1}{2} Q^2}{C^2} = \frac{1}{2} Q^2 \left(\frac{1}{C'^2} - \frac{1}{C^2}\right)$

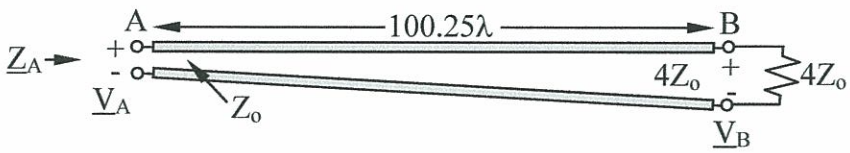
$= \frac{1}{2} Q^2 \left(\left(\frac{\varepsilon_0 d}{10 bL}\right)^2 - \left(\frac{\varepsilon_0 d}{bL}\right)^2\right)$

$= -\frac{1}{2} Q^2 (.99) \frac{\varepsilon_0 d}{bL}$

$= -\frac{.99}{2} \frac{Q^2 \varepsilon_0 d}{bL}$

Problem 2. (20/200 points)

The plate separation of a lossless parallel-plate TEM line many wavelengths long (length D = 100.25λ) very slowly increases from end A to end B, as illustrated. This increases the characteristic impedance of the line from Z_o at the input end A, to $4Z_o$ at the output end B. This transition from A to B is so gradual that it produces no reflections. End B is terminated with a resistor of value $4Z_o$



a) What is the input impedance $\underline{Z}_A$ seen at end A? Explain briefly.

$\underline{Z}_A = Z_o$

Explanation:

Because impedence is voltage/current the input voltage/current ratio is Z_o and there is no reflected wave to reduce the input voltage.

10/10

b) If the sinusoidal (complex) input input voltage is $\underline{V}_A$, what is the output voltage $\underline{V}_B$?

Since impedences are no reflections at junction = 0 b/c $Z_o(z) = Z_L$

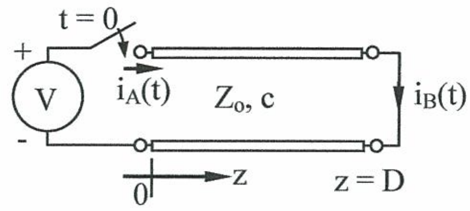
$\underline{V}_B = \cancel{\underline{V}_A} \quad -2j\underline{V}_a$

0/10

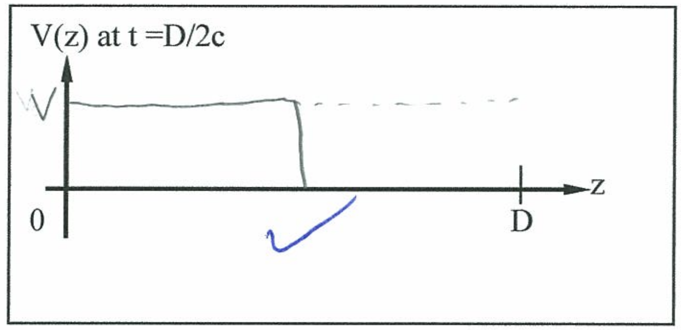
Please turn sheet over for Problem 3.

Problem 3. (25/200 points)

At t = 0 a switch connects a voltage V to a passive air-filled short-circuited TEM line of length D and characteristic impedance Z_o, as illustrated. Please sketch and quantify dimension:

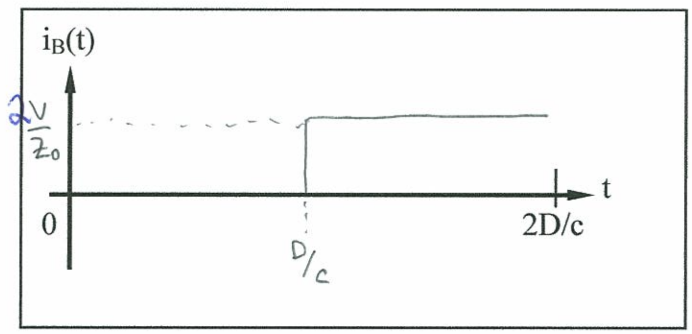


a) The line voltage v(z) at t = D/2c.



8/8

b) The current $i_B(t)$ through the short circuit for 0 < t < 2D/c.

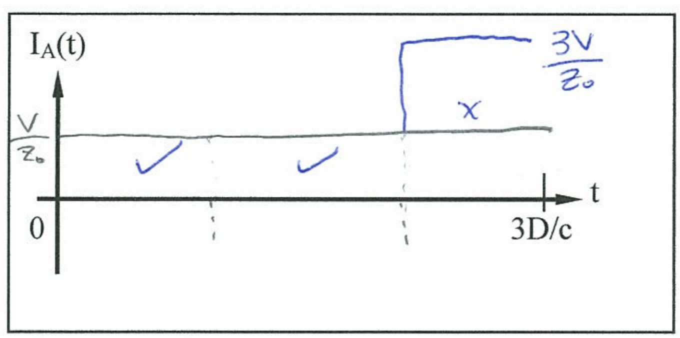


$\Gamma_L = -1$

4/8

The time axis is correct but I deducted half marks here for forgetting the factor of 2

c) The current $i_A(t)$ from the voltage source (z = 0) for 0 < t < 3D/c.

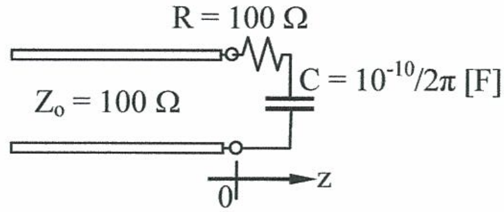


5/8

$\frac{V}{Z_o}$

~~Sho~~ this could have qualified as a carry fwd since it is intrinsically linked to (b), however, I deducted marks for omitting the same current jump as in (b) in the last 1/3rd of the time series

Problem 4. (30/200 points)



A 100-ohm air-filled lossless TEM line is terminated with a 100-ohm resistor and a $10^{-10}/2\pi$ Farad capacitor in series, as illustrated. It is driven at 100 MHz.

a) What fraction $F = |\underline{\Gamma}_L|^2$ of the incident power is reflected from this load?

$\underline{F} = \quad \frac{1}{\sqrt{104}} \quad \frac{1}{5}$

$$Z_L = 100 + \frac{1}{j\omega C}$$

$$= 100 + \frac{1}{j\,\cancel{10^7}\cdot 10^{-10}}$$

$$= 100 - j\,100\cancel{0}$$

$$Z_n = 1 - j$$

5/10

The math here is correct, but I made a simple mistake of converting 100 MHz to 10^7 instead of 10^8

~~10~~ 108 CARRY FWD ERROR (USED 10 MHz instead of 100)

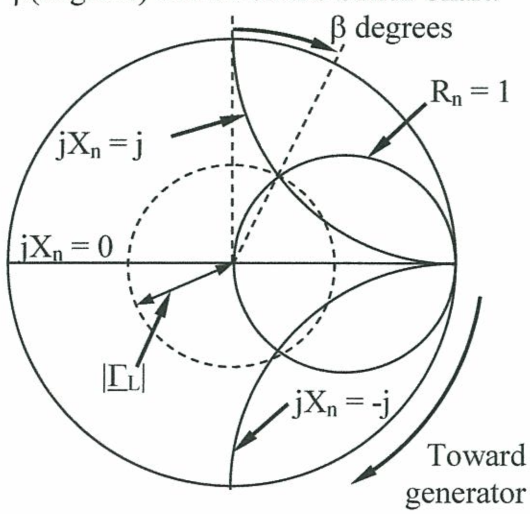
$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{(100 - j1000) - 100}{100 - j1000 + 100}$$

$$= \frac{-j1000}{200 - j1000}$$

$$|\Gamma_L|^2 = \frac{1000}{|200 - j1000|^2} = \frac{1000}{\sqrt{40000 + 1000000}}$$

$$= \frac{1000}{\sqrt{104}\cdot 10^3}$$

b) What is the minimum distance D(meters) from the load at which the line current $|\underline{I}(z)|$ is maximum? You may express your answer in terms of the angle β(degrees) shown on the Smith Chart.



D_{min} = $3\cdot 10^{-2}(\beta)$

X

*

$$i(z) = i_+ - i_-$$

0/10

Please turn sheet over to answer part (c).

c) Can we match this load by adding another capacitor in series somewhere and, if so, at what distance D and with what value C_m?

Can we match? YES ~~NO~~
D = $7.5 \cdot 10^{-3}$ m ✗ $3/4 + \beta/120$
$C_m = \frac{10^{-10}}{2\pi}$ F ✓

7/10

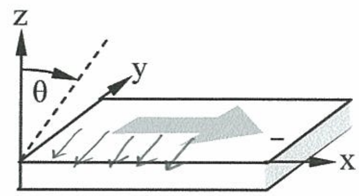
This is essentially a rounding of 2/3 of the 10 pts. for this question.

$\lambda = \frac{f}{c} = \frac{10^7}{3 \cdot 10^8} = \frac{1}{30}$ m $3 \cdot 10^{-2}$ m

$\lambda/4 = 7.5 \cdot 10^{-3}$

Problem 5. (20/200 points)

A flat perfect conductor has a surface current in the xy plane at z = 0 of:



$$\overline{J}_s = \hat{x} J_o e^{-jbx} \text{ [A/m].}$$

a) Approximately what is $\overline{H}$ in the xy plane at z = 0+?

$J_o e^{-jbx}$

$\overline{H}(z = 0+) =$

$-\hat{y} J_o e^{-jbx}$

10/10

b) How might one easily induce this current sheet at frequency f [Hz] on the surface of a good conductor? Please be reasonably specific and quantitative.

By applying an EM plane wave of frequency f to the surface, which has an x-component wavenumber b and an $\overline{H}$ in the $\hat{y}$ direction we could induce this current sheet.

5/10 (not enough detail) Again, correct, but I didn't include the magnitude of the wave or its formula

To induce this current one might:

Please turn sheet over for Problem 6.

Problem 6. (10/200 points)

A certain evansescent wave at angular frequency ω in a slightly lossy medium has $\underline{\bar{E}} = \hat{y} E_o e^{\alpha(x-0.01z) - jbz}$; assume $\mu = \mu_o$. What is the distance D between phase fronts for this wave?

D = $\frac{2\pi}{b}$

$e^{\alpha(x-0.01z)-jbz}$

decay along $\hat{x}$ and $-\hat{z}$ direction

wave character along z direction

will be maximum at values of

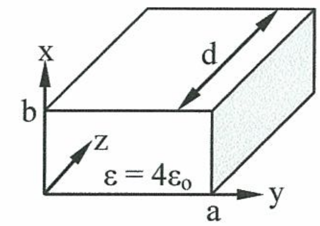
$bz = n2\pi$

$D = \frac{2\pi}{b}$

✓

10/10

Problem 7. (25/200 points)



A resonator is filled with a dielectric having $\varepsilon = 4\varepsilon_o$ and has dimensions b, a, and d along the x, y, and z directions, respectively, where d > a > b.

a) What is the lowest resonant frequency $f_{m,n,q}$ [Hz] for this resonator?

$f_{m,n,q}$ [Hz] =

$$\frac{c}{4q^2}$$

* ignoring DC "resonant" frequency

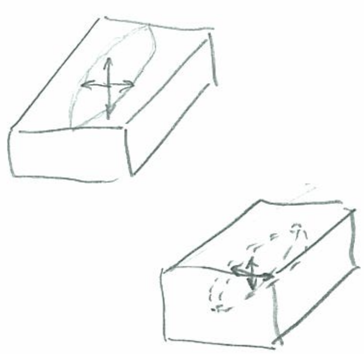
$$f_{mnq} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{q}{d}\right)^2}$$

$$\cancel{f_{001}} = \frac{c}{4}\left(\frac{1}{b^2}\right) =$$

f_{101}

0/8

b) What is the polarization of the electric vector $\bar{E}$ at the center of the resonator for this lowest frequency mode?



Polarization of $\bar{E}$ is:

linear in $\hat{z}$ direction

CARRY FWD

4/8

half marks for correctly identifying linear polarization, but wrong axis (again, could be a carry fwd from (a) but I deducted anyways)

Please turn sheet over to answer part (c).

c) What is the Q of this resonance if the dielectric has a slight conductivity σ? Hint: a ratio of integrals may suffice, so the integrals might not need to be computed.

$$\boxed{Q = \frac{\frac{c}{4q^2}}{} \quad \frac{c\varepsilon_0}{8\pi\sigma q^2}}$$

$$Q = \frac{\omega_0 W_T}{P_{diss}} = \frac{\frac{2\pi f_{101}}{\cancel{2\pi}} \cdot \int_V \frac{\varepsilon|E|^2}{4} dv}{\int_V \sigma |E|^2 dv} = \frac{f_{001}\,\varepsilon}{8\pi\,\sigma} = \frac{\frac{c}{4q^2}4\varepsilon_0}{8\pi\sigma}$$

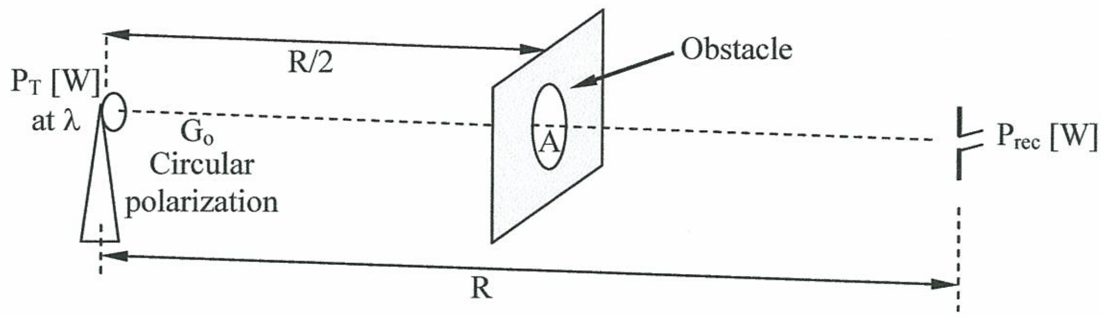
$$P_{diss} = I^2 R = \int (\sigma E)^2 \frac{1}{\cancel{\sigma}}$$

9/9

CARRY FWD ERROR (WRONG HARMONIC FROM (A))

Problem 8. (20/200 points)

A certain transmitter transmits P_T watts of circularly polarized radiation with antenna gain G_o (in circular polarization) toward an optimally oriented matched short-dipole receiving antenna (gain = 1.5) located a distance R away. The wavelength is λ.



a) In the absence of any obstacles or reflections, what power P_R is received?

$$P_R = \frac{1.5 P_T G_o \lambda^2}{4\pi R^2}$$

$$\frac{P_T G_o}{4\pi R^2}$$

3

power available at receiving antenna

$$\frac{\cancel{1}.5 P_T G_o}{2 \cdot 4 \pi R^2}$$

0/10

Please turn sheet over to answer part (b).

b) A large metal fence is then erected half way between the transmitter and receiver, and perpendicular to the line of sight. Fortunately it has a round hole of area A centered on that line of sight. Assume the hole is sufficiently small that the electrical phase of the incident wave is constant over its entirety. What power is received now?

$$P_R = \frac{1.5\left(\frac{AP_TG_o}{\pi R^2}\right)}{\pi R^2}$$

0/10

$P_{rec_{fence}} \doteq P_{RF}$

$= \frac{AP_TG_o}{4\pi R^2}$

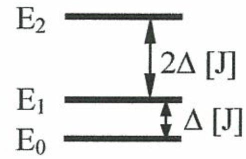
$P_{TF} =$

gain here will be a sinc function of θ which at $\theta = 0$ = 1

5/16/09

Problem 9. (15/200 points)

An ideal lossless three-level laser has the illustrated energy level structure. Level 1 is Δ Joules above the ground state, and Level 2 is 3Δ Joules above the ground state. All rates of spontaneous emission A_{ij} have the same finite value except for A_{21}, which is infinite.



a) What should be the laser frequency f_L [Hz]?

f_L [Hz] = $\frac{\Delta}{h}$

Δ joules $= \hbar f$

$f =$

7/7

b) What is this laser's maximum possible efficiency η = (laser power)/(pump power)?

η = $\frac{1}{3}\eta_0$

constant which depends on the rate of spontaneous emission

$\eta = \left(\frac{1}{3}\right)($

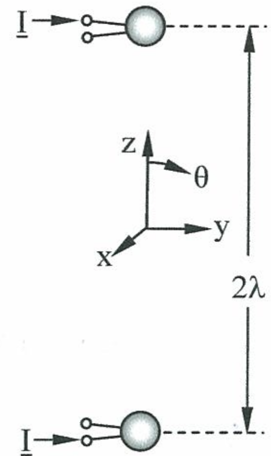
8/8

$\eta_0 \propto A_{ij}$

Please turn sheet over for Problem 10.

Problem 10. (10/200 points)

Two monopole (isotropic) acoustic antennas lying on the z axis are aligned in the z direction and separated by 2λ, as illustrated. <u>They are fed 180° out of phase</u>. In what directions θ does this acoustic array have maximum gain $G(\theta)$? Simple expressions suffice. If more than one direction has the same maximum gain, please describe all such directions.

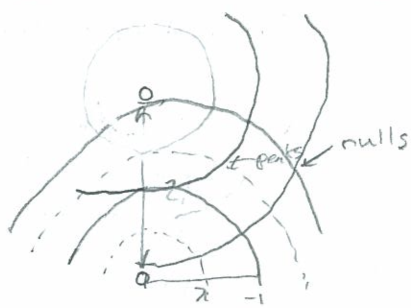


$\theta =$ $\pi/4$, $3\pi/4$, $5\pi/4$, $7\pi/4$

have maximum gain.

should be $\pm\cos^{-1}(1/4)$, $\pm\cos^{-1}(3/4)$ not $\pi/4$, $3\pi/4$

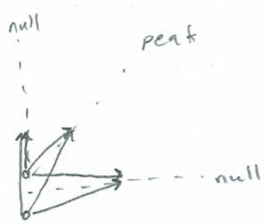
Gain will be maximum when both wave fronts are in phase



$\cos^2\left(\frac{dk}{2}\sin\theta + \alpha\right)$

$\cos^2\left(\frac{2\lambda\frac{2\pi}{\lambda}}{2}\sin\theta + \alpha\right)$

0/10



5/16/09