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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.013 Electromagnetics and Applications

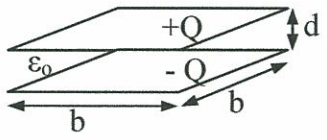
Student Name:

Final Exam Closed book, no calculators

Please note the two pages of formulas provided at the back; the laser and acoustic expressions have been revised slightly. There are 10 problems; some are on the back sides of the sheets. For full credit, please ***simplify all expressions****, present* ***numerical answers to the extent practical*** *without a calculator or tedious computation, and place your* ***final answers within the boxes provided****. You may leave natural constants and trigonometric functions in symbolic form (π, ε_o, μ_o, η_o, h, e, $\sin(0.9)$, $\sqrt{2}$, etc.). To receive partial credit, provide all related work on the same sheet of paper and give brief explanations of your answer. Spare sheets are at the back.*

Problem 1. (25/200 points)

Two square capacitor plates in air have separation d, sides of length b, and charge ±Q as illustrated. Fringing fields can be neglected.



a) What is the capacitance C_a of this device?

$C_a = \dfrac{\varepsilon_o b^2}{\varepsilon_o d}$

$C = \dfrac{Q}{V}$

$E_o = \dfrac{Q\varepsilon_o}{b^2}$

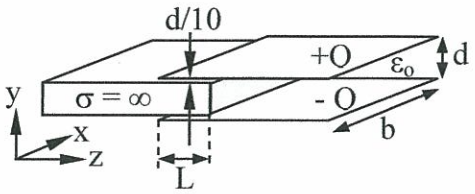
$V = E_o d = \dfrac{Q\varepsilon_o d}{b^2}$

$C = \dfrac{Q}{\frac{Q\varepsilon_o d}{b^2}} = \dfrac{b^2}{\varepsilon_o d}$

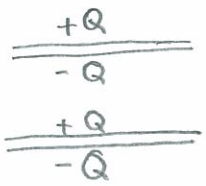
4/8 NOTE: I deducted half marks for flipping ε_o

Please turn sheet over to answer parts (b) and (c).

b) A perfectly conducting plate is introduced between the capacitor plates, leaving parallel gaps of width d/10 above and below itself. What now is the device capacitance C_b when it is fully inserted?



$C_b =$ $\dfrac{b^2}{\varepsilon_0 d}$



Since there cannot be charge within the interior a perfect conductor the result is to maintain the same charge over a smaller distance

$C = \dfrac{b^2}{\varepsilon_0 (\frac{1}{10}) d}$

4/8

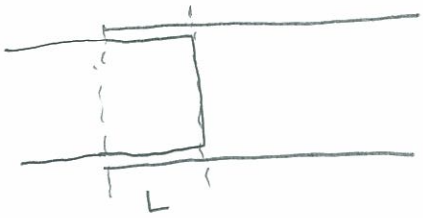
Here I deducted half marks for forgetting the ½ factor of series capacitors

c) What is the magnitude and direction of the force $\bar{f}$ on the new plate of Part (b) as a function of the insertion distance L. Please express your answer as a function of the parameters given in the figure.

$\bar{f} =$ $-.45 \dfrac{Q^2 \varepsilon_0 d}{bL^2}$

Using the energy method,

$W_e = \frac{1}{2} C V^2$



$W_e = \frac{1}{2} C V^2$ X 0/9

$= \frac{1}{2} C \left(\frac{Q}{C}\right)^2$

$= \frac{1}{2} \frac{Q^2}{C}$

$F = \frac{W_e}{L}$

$\Delta W_e = \frac{1}{2}\frac{Q^2}{C'} - \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}Q^2\left(\frac{1}{C'} - \frac{1}{C}\right)$

$= \frac{1}{2}Q^2\left(\left(\frac{\varepsilon_0 d}{10bL}\right) - \left(\frac{\varepsilon_0 d}{bL}\right)\right)$

$= -\frac{1}{2}Q^2(.99)\frac{\varepsilon_0 d}{bL}$

$= -\frac{.99}{2}\frac{Q^2 \varepsilon_0 d}{bL}$

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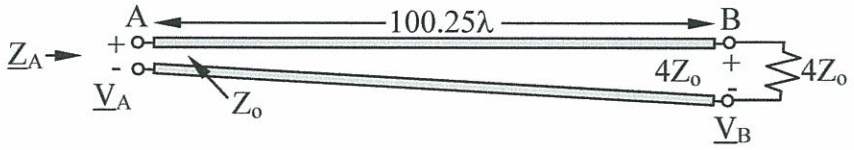
Problem 2. (20/200 points)

The plate separation of a lossless parallel-plate TEM line many wavelengths long (length D = 100.25λ) very slowly increases from end A to end B, as illustrated. This increases the characteristic impedance of the line from Z_o at the input end A, to $4Z_o$ at the output end B. This transition from A to B is so gradual that it produces no reflections. End B is terminated with a resistor of value $4Z_o$



a) What is the input impedance $\underline{Z}_A$ seen at end A? Explain briefly.

$\underline{Z}_A = Z_o$

Explanation:

Because impedence is voltage/current the input voltage/current ratio is Z_o and there is no reflected wave to reduce the input voltage.

10/10

b) If the sinusoidal (complex) input input voltage is $\underline{V}_A$, what is the output voltage $\underline{V}_B$?

$\underline{V}_B = \underline{V}_A - 2j\underline{V}_a$

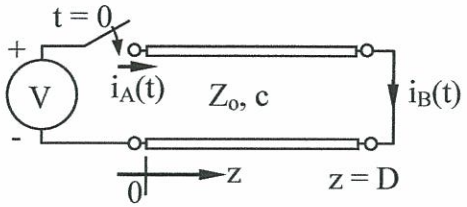
Since impedences are matched there are no reflections at junction $\Gamma = 0$ b/c $Z_o(z) = Z_L$

0/10

Please turn sheet over for Problem 3.

Problem 3. (25/200 points)

At t = 0 a switch connects a voltage V to a passive air-filled short-circuited TEM line of length D and characteristic impedance Z_o, as illustrated. Please sketch and quantify dimension:



a) The line voltage v(z) at t = D/2c.

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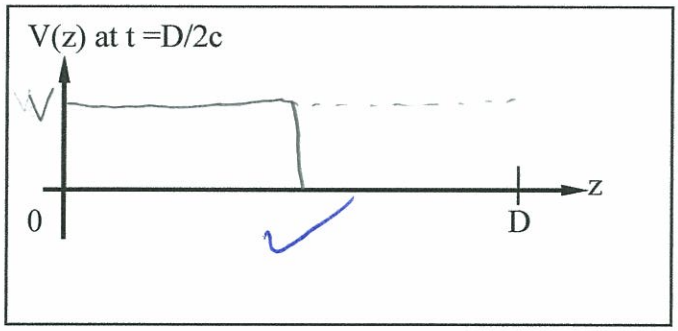

b) The current $i_B(t)$ through the short circuit for $0 < t < 2D/c$.

$\Gamma_L = -1$

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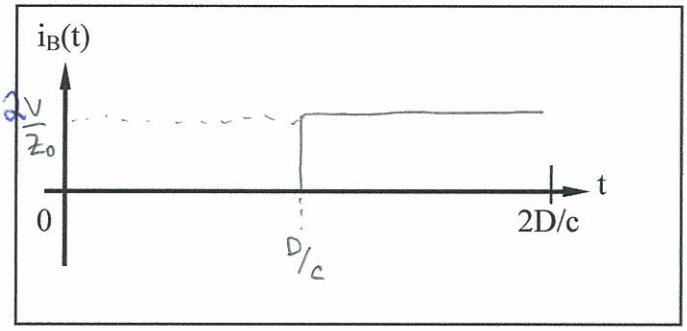
The time axis is correct but I deducted half marks here for forgetting the factor of 2

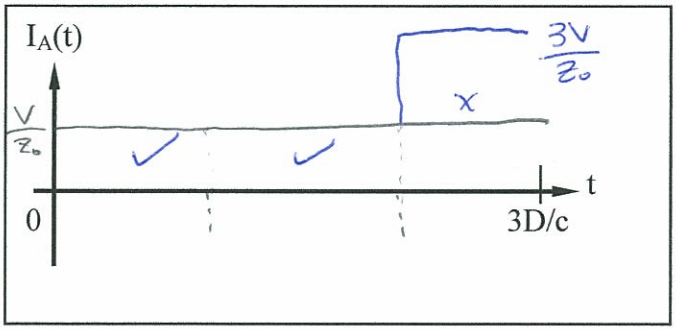


c) The current $i_A(t)$ from the voltage source (z = 0) for $0 < t < 3D/c$.

5/8

$\frac{V}{Z_o}$

This could have qualified as a carry fwd since it is intrinsically linked to (b), however, I deducted marks for omitting the same current jump as in (b) in the last 1/3rd of the time series



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