

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
 Department of Electrical Engineering and Computer Science
6.013 Electromagnetics and Applications

Student Name:

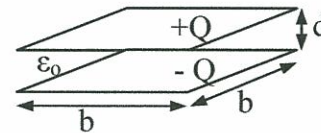
Final Exam

Closed book, no calculators

Please note the two pages of formulas provided at the back; the laser and acoustic expressions have been revised slightly. There are 10 problems; some are on the back sides of the sheets. For full credit, please **simplify all expressions**, present **numerical answers to the extent practical** without a calculator or tedious computation, and place your **final answers within the boxes provided**. You may leave natural constants and trigonometric functions in symbolic form (π , ϵ_0 , μ_0 , η_0 , h , e , $\sin(0.9)$, $\sqrt{2}$, etc.). To receive partial credit, provide all related work on the same sheet of paper and give brief explanations of your answer. Spare sheets are at the back.

Problem 1. (25/200 points)

Two square capacitor plates in air have separation d , sides of length b , and charge $\pm Q$ as illustrated. Fringing fields can be neglected.



a) What is the capacitance C_a of this device?

$$C = \frac{Q}{V}$$

$$E_0 = \frac{QE_0}{b^2}$$

$$V = E_0 d = \frac{QE_0 d}{b^2}$$

$$C_a = \frac{\epsilon_0 b^2}{\epsilon d}$$

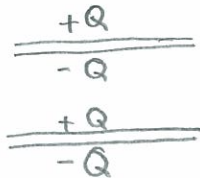
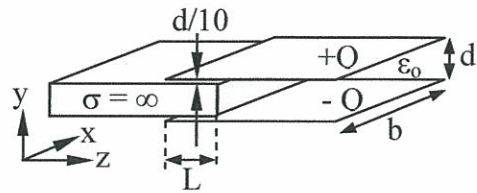
$$C = \frac{Q}{\frac{QE_0 d}{b^2}} = \frac{b^2}{\epsilon_0 d}$$

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NOTE: I deducted half marks for flipping ϵ_0

Please turn sheet over to answer parts (b) and (c).

b) A perfectly conducting plate is introduced between the capacitor plates, leaving parallel gaps of width $d/10$ above and below itself. What now is the device capacitance C_b when it is fully inserted?



$$C_b = \frac{5b^2}{\epsilon_0 d}$$

Since there cannot be charge within the interior of a perfect conductor the result is to maintain the same charge over a smaller distance

Here I deducted half marks for forgetting the $1/2$ factor of series capacitors

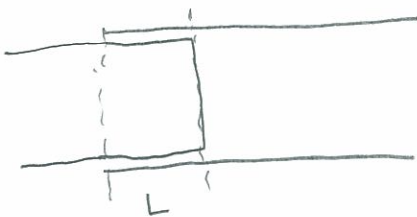
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c) What is the magnitude and direction of the force \bar{f} on the new plate of Part (b) as a function of the insertion distance L . Please express your answer as a function of the parameters given in the figure.

Using the energy method,

$$W_e = \frac{1}{2} CV^2$$

$$\bar{f} = -0.45 \frac{Q^2 \epsilon_0 d}{bL^2}$$



$$W_e = \frac{1}{2} CV^2$$

x 9/9

$$= \frac{1}{2} C \left(\frac{Q}{C} \right)^2$$

$$= \frac{1}{2} \frac{Q^2}{C}$$

$$F = \frac{W_e}{L}$$

$$\Delta W_e = \frac{1}{2} \frac{Q^2}{C^{1/2}} - \frac{1}{2} \frac{Q^2}{C^2} = \frac{1}{2} Q^2 \left(C^{-1/2} - C^{-2} \right)$$

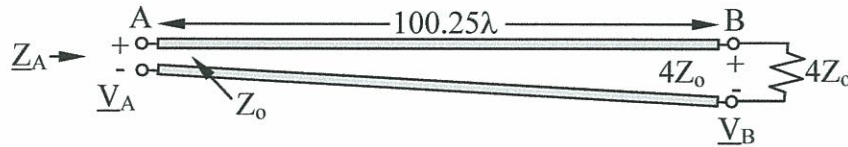
$$= \frac{1}{2} Q^2 \left(\left(\frac{\epsilon_0 d}{10bL} \right)^2 - \left(\frac{\epsilon_0 d}{5bL} \right)^2 \right)$$

$$= -\frac{1}{2} Q^2 (0.99) \frac{\epsilon_0 d}{bL^{5/16/09}}$$

$$= -\frac{0.99}{2} \frac{Q^2 \epsilon_0 d}{bL}$$

Problem 2. (20/200 points)

The plate separation of a lossless parallel-plate TEM line many wavelengths long (length $D = 100.25\lambda$) very slowly increases from end A to end B, as illustrated. This increases the characteristic impedance of the line from Z_0 at the input end A, to $4Z_0$ at the output end B. This transition from A to B is so gradual that it produces no reflections. End B is terminated with a resistor of value $4Z_0$.



a) What is the input impedance \underline{Z}_A seen at end A? Explain briefly.

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$\underline{Z}_A = Z_0$

Explanation:
 Because impedance is voltage/current the input voltage/current ratio is Z_0 and there is no reflected wave to reduce the input voltage.

b) If the sinusoidal (complex) input input voltage is \underline{V}_A , what is the output voltage \underline{V}_B ?

Since impedances are the same there are no reflections at junction $\Gamma = 0$
 b/c $Z_0(z) = Z_L$

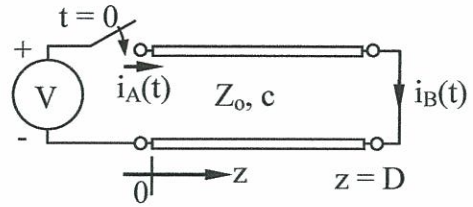
$\underline{V}_B = \underline{V}_A - 2j \underline{V}_A$

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Please turn sheet over for Problem 3.

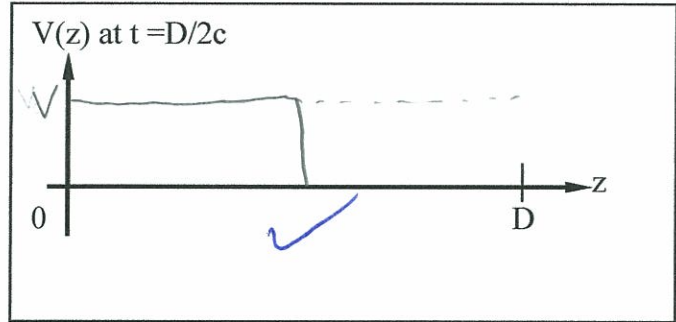
Problem 3. (25/200 points)

At $t = 0$ a switch connects a voltage V to a passive air-filled short-circuited TEM line of length D and characteristic impedance Z_0 , as illustrated. Please sketch and quantify dimension:



a) The line voltage $v(z)$ at $t = D/2c$.

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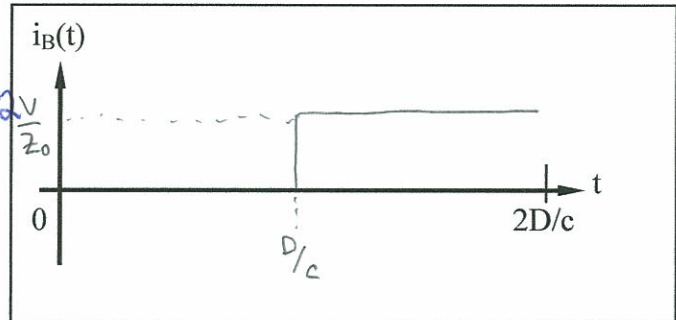


b) The current $i_B(t)$ through the short circuit for $0 < t < 2D/c$.

$$\Gamma_L = -1$$

4/8

The time axis is correct but I deducted half marks here for forgetting the factor of z



c) The current $i_A(t)$ from the voltage source ($z = 0$) for $0 < t < 3D/c$.

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$$\frac{V}{Z_0}$$

~~But~~ This could have qualified as a carry fwd since it is intrinsically linked to (b), however, I deducted marks for omitting the same current jump as in (b) in the last 1/3rd of the time series

