Now we have two functions

\[ \ddot{x} + \frac{k}{m} x = 0 \]

\[ x = A \cos(\omega t + \phi) \]

Take the derivative of this

\[ \dot{x} = -A \omega \sin(\omega t + \phi) \]

Now take the derivative again

\[ \ddot{x} = -A \omega^2 \cos(\omega t + \phi) = -A \omega^2 \cos(\omega t + \phi) \]

\[ -\omega^2 x = \frac{k}{m} x = 0 \]

\[ \omega = \sqrt{\frac{k}{m}} \]
Understanding a Simple Harmonic Oscillator

**STARTING POINT:**

\[ \text{FORCE} = \text{POSITION} \]

\[ M \ddot{x} = -kx \]

1. \( \dot{x} \) is really \( \dddot{x} \)

2. \( \dddot{x} = -kx \)

\[ \frac{\dddot{x}}{m} = \frac{k}{m} x \]

**Goal 1:**

get the function into the terms

\[ \dddot{x} + \frac{k}{m} x = 0 \]

This is a SHO because its acceleration is exactly opposite to its position.

Next we compare this to an *invented* function which also describes the motion:

\[ x = A \cos(wt + \phi) \]

This is *always* the case for SHO's.

Now for the tricky bit...

**Goal 2:**

we need to get \( \omega \) in relation to the first goal's function

This is possible because \( \dddot{x} \) is in both equations.

We can do this by taking the double derivative of the second equation which creates beautiful cancellations, greatly simplifying the math.

\[ \dddot{x} = -A \omega \sin(wt + \phi) \]
\[ \dddot{x} = -A \omega^2 \cos(wt + \phi) \]
Now we have the equation in terms of acceleration:

\[ \ddot{x} = -A\omega^2 \cos(\omega t + \phi) \]

Note the symmetry between \( \ddot{x} \) and \( x \) however. They are *almost* the same function.

\[ x = A \cos(\omega t + \phi), \quad \ddot{x} = -A\omega^2 \cos(\omega t + \phi) \]

In fact, let's rearrange them to separate the terms:

\[ x = A \cos(\omega t + \phi) \]

\[ \ddot{x} = A \cos(\omega t + \phi) \cdot (-\omega^2) \]

This is why the period is so easy for such a complex calculation.

Finally, we substitute \( \ddot{x} \) for \( x \) in our final equation:

\[ -\omega^2 x = -\frac{k}{m} x \]

Finally, since \( \omega \) is defined by:

\[ T = \frac{2\pi}{\omega} \]

this also gives us the period for free:

\[ T = 2\pi \sqrt{\frac{m}{k}} \]
SHO: test case

Pendulum of odd mass/shape

\[ \vec{F} = \vec{F} \times mg \]
\[ \vec{P}_B = rmg \sin \theta \]
1) FORCE ↔ TRANSFORMED POSITION
2) ACCELERATION

\[ I \alpha = rmg \sin \theta \]
\[ I \ddot{\theta} = rmg \sin \theta \]

NOW 2nd EQUATION

\[ \theta = A \cos(\omega t + \phi) \]
\[ \dot{\theta} = -A \cos(\omega t + \phi) \omega \]
\[ \ddot{\theta} = -A \cos(\omega t + \phi) \omega^2 \]
\[ \ddot{\theta} = -\omega^2 \theta \]

ALWAYS THE RESULT

\[ -I \omega^2 \dot{\theta} = rmg \sin \theta \]

\[ I \omega^2 = rmg \sin \theta \]

\[ \omega = \sqrt{\frac{rmg \sin \theta}{I}} \]

Because the angles are small, I can cancel these

\[ \omega = \sqrt{\frac{rmg}{I}} \]

\[ T = 2\pi \sqrt{\frac{I}{rmg}} \]
Understanding Torque and Angular Momentum

This is a torque force. It is like a corkscrew (lefty-loosy, righty-tighty!)

IF a force is applied, ask where torque is zero

Why is this?

Well examine any other point and ask how much twisting force is being applied?

It's like the earth is twisting with the same momentum at every point

\[ \alpha_1 m_1 = \alpha_2 m_2 \]

Angular velocity is a constant, but because the size of the orbit changes velocity changes
TORQUE ≠ FORCE

Consider

Angular Spin momentum occurs when spin is conserved around the center of mass
Understanding Bernoulli's Equation

\[ \frac{1}{2} \rho v^2 + \rho g y + P_0 = C \]

What Bernoulli is saying is that if you were to add up the total energy created by movement, gravity, and pressure, this would be a constant.

Of course, because unless temperature changes, where is the energy coming from?

As you push water through a smaller area, its speed changes proportionally.

**Continuity Equation:**

\[ v_1 A_1 = v_2 A_2 \]

Pascal's Law says that the pressure increases linearly with depth. (Imagine the extra weight of the water straight up added to each point)

**Pascal's Law**

\[ \text{Weight/A} = \Delta \text{Pressure} \]

Here the bottom will be flowing faster for 3 reasons:

1. The area is smaller so there is more force
2. Gravity has been changed into energy
3. Pressure basic has increased by:

\[ p_1 - p_2 = \rho g (y_2 - y_1) \]

(Pascal's Law)