Now we have two functions

$$\ddot{x} + \frac{k}{m}x = 0 \quad \boxed{\&} \quad x = A\cos(\omega t + \varphi)$$

take the derivative of this

$$\dot{x} = -A\omega\sin(\omega t + \varphi)$$

now take the derivative again

$$\ddot{x} = -A\omega^2\cos(\omega t + \varphi) = \cancel{A\omega} -\omega^2 A\cos(\omega t + \varphi)$$

equivalent $\therefore$

$$= \boxed{-\omega^2 x}$$

$$= \cancel{A\omega^2\sin(\omega t + \varphi)}$$

TAKE THIS AND PLUG INTO EQUATION

$$\underbrace{\ddot{x}}_{-\omega^2 x} + \frac{k}{m}x = 0$$

$$-\omega^2 x + \frac{k}{m}x = 0$$

$$\omega^2 \cancel{x} = \frac{k}{m}\cancel{x}$$

$$\omega = \sqrt{\frac{k}{m}}$$

Understanding a Simple Harmonic Oscillator

$\boxed{\text{STARTING POINT}}$

FORCE = POSITION (TRANSFER)

$ma = -kx$

1: a is really $\ddot{x}$

$\therefore$ $m\ddot{x} = -kx$

$$\frac{m\ddot{x}}{m} = \frac{-kx}{m}$$

$$\boxed{\ddot{x} + \frac{k}{m}x = 0}$$

$\boxed{\text{Goal 1:}}$ get the function into the terms

$$\underbrace{\ddot{x} + ?\,x = 0}$$

this is a SHO because its acceleration is exactly opposite to its position

Next we compare this to an <u>invented</u> function which <u>ALSO</u> describes the motion:

$$\boxed{x = A\cos(\omega t + \varphi)}$$

This is <u>ALWAYS</u> the case for <u>SHO's!!</u>

Now for the tricky bit...

$\boxed{\text{Goal 2:}}$ We need to get ω in relation to the first goal's function. This is possible because $x/\ddot{x}$ is in both equations.

We can do this by taking the double derivative of the second equation which creates beautiful cancellations, greatly simplifying the math

$$\dot{x} = -A\omega\sin(\omega t + \varphi)$$

$$\ddot{x} = -A\omega^2\cos(\omega t + \varphi)$$

Now we have the equation in terms of acceleration.

$$\underbrace{\ddot{x}}_{\text{acceleration}} = -A\omega^2\cos(\omega t + \varphi)$$

acceleration equals amplitude frequency squared! function of periodicity

Note the symmetry between x and $\ddot{x}$ however. They are ALMOST the same function.

$$x = A\cos(\omega t + \varphi)$$
$$\ddot{x} = -A\omega^2\cos(\omega t + \varphi)$$

In fact, lets rearrange them to separate the terms

$$x = A\cos(\omega t + \varphi)$$
$$\ddot{x} = \underbrace{A\cos(\omega t + \varphi)}_{x} \cdot (-\omega^2)$$

$$\ddot{x} = x \cdot (-\omega^2) \boxed{= -\omega^2 x}$$

This is why the period is so easy for such a complex calculation

Finally we substitute $\ddot{x}$ for in our final equation

$$-\omega^2 x = -\frac{k}{m}x$$
$$\Downarrow$$
$$\omega^2 = \frac{k}{m}$$
$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

Finally since period is BY DEFINITION

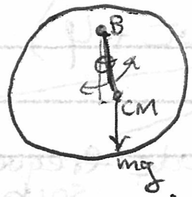
$$T = \frac{2\pi}{\omega}$$

this also gives us the period for free

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

SHO: test case

Pendulum of odd mass/shape



$$\tau_B = \vec{r} \times \vec{mg}$$

$$\tau_B = rmg\sin\theta$$

1) FORCE ↔ TRANSFORMED POSITION

2) ACCELERATION " "

$$I\alpha = rmg\sin\theta$$

$$I\ddot{\theta} = rmg\sin\theta$$

NOW 2nd EQUATION

$$\theta = A\cos(\omega t + \varphi)$$

$$\dot{\theta} = -A\cos(\omega t+\varphi)\omega$$

$$\ddot{\theta} = -A\cos(\omega t + \varphi)\omega^2$$

$$\boxed{\ddot{\theta} = -\omega^2\theta}$$ ← ALWAYS THE RESULT

LAW OF SMALL angles

$$\boxed{\begin{array}{l}\theta \doteq \sin\theta \\ \theta \doteq 1-\cos\theta\end{array}}$$

$$-I\omega^2\theta = rmg\sin\theta$$

$$I\omega^2\theta = rmg\sin\theta$$

$$\frac{\sin x}{x} =$$

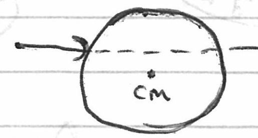
$$\omega = \sqrt{\frac{rmg\cancel{\sin\theta}}{I\cancel{\theta}}}$$ ← Because the angles are small, I can cancel these

$$\boxed{\omega = \sqrt{\frac{rmg}{I}}} \therefore \boxed{T = 2\pi\sqrt{\frac{I}{rmg}}}$$

Understanding Torque and Angular Momentum

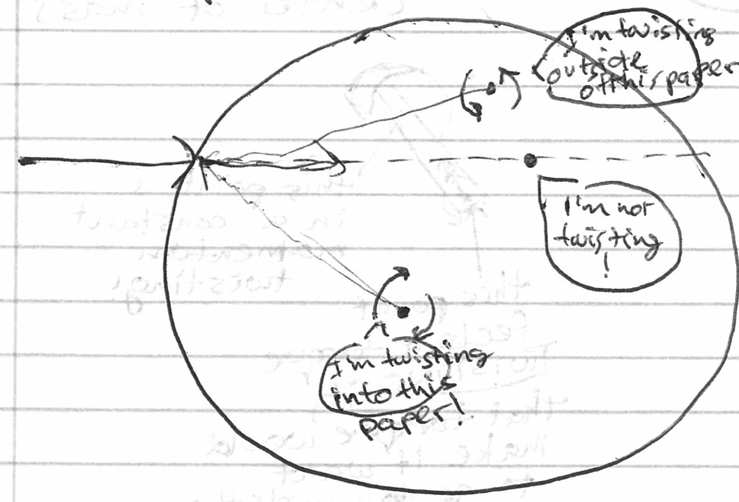
This is a torque force. It is like a corkscrew (lefty loosy/ righty-tighty!)

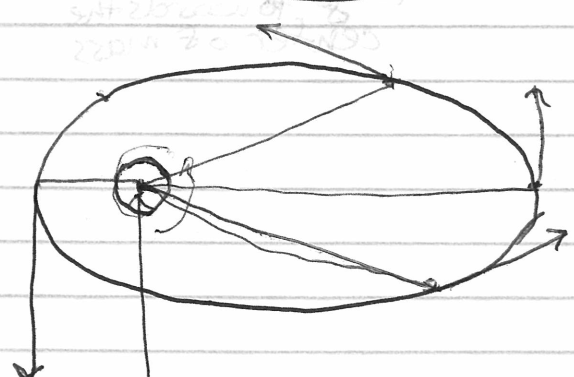
IF a force is applied, ask where torque is zero



why is this?

Well examine any other point and ask how much **TWISTING** force is being applied?





It's like the earth is twisting with the same momentum at every point

$\omega_1 m_1 = \omega_2 m_2$

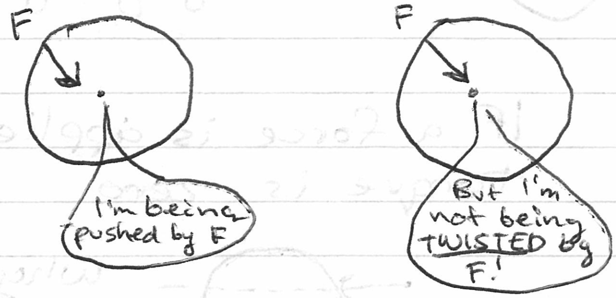
angular velocity is a const.

but because the size of the orbit changes velocity changes

angular momentum is conserved here at this point, why?

TORQUE ≠ FORCE

Consider

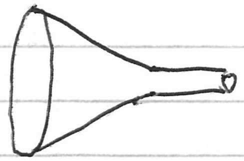


Angular Spin momentum occurs when spin is conserved around the center of mass

this point is in a constant momentum twisting!

this point feels a torque TWISTING it, that torque would make it want to go towards the center of mass

Understanding Bernoulli's Equation



$$\tfrac{1}{2}\rho v^2 + \rho g y + P_y = C$$

What B is saying is that if you were to add up the total energy created by movement, gravity and pressure → this would be a constant.

Of course, because unless temperature changes, where is the energy coming from?

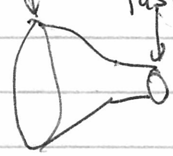
CONTINUITY EQUATION says:

→ as you push water through a smaller area its speed changes proportionally

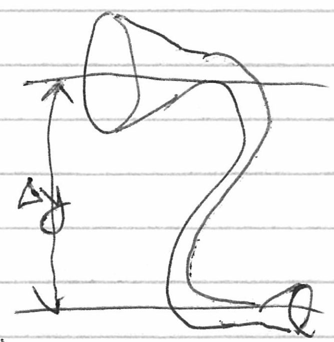
$$V_1 A_1 = V_2 A_2$$

this has higher pressure

faster b/c of this law



BUT BECAUSE OF BERNOULLI THAT MEANS



here the bottom will be flowing faster for 3 reasons

1) the area is smaller so there is more force

2) Gravity has been changed into energy

3) Pressure basic has incre[a]sed by: $P_1 - P_2 = \rho g (y_2 - y_1)$ (Pascal's Law)

Pascal's Law says that the pressure increases linearly with depth. (imagine the extra weight of the water straight up added to each point)