~~12~~ 14

$\boxed{75.1°}$

Problem 1 (16 points)

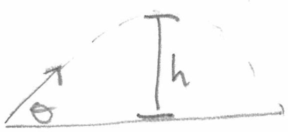
A gunner fires a bullet of mass m with speed v_0 at an angle θ from the horizontal plane. Assume the bullet is fired from ground level. The gravitational acceleration is g. Ignore air drag. Express your answers in terms of m, g, v_0, and θ.

4 a. (4) When does the bullet reach its highest point?

$\boxed{2}$ b. (4) How high is that above the ground?

4 c. (4) With what speed will the bullet hit the ground?

4 ~~2~~ d. (4) What is the horizontal distance that the bullet has traveled when it hits the ground?



a) $\dot{y} = v_0 \sin\theta$

$v_f = v_i + at$

$0 = \dot{y} - \cancel{m}gt$

$\dot{y} = \cancel{m}gt$

$t = \frac{\dot{y}}{\cancel{m}g}$

$\boxed{t = \frac{v_0 \sin\theta}{\cancel{m}g}}$

(2) NO MASS IN ACCEL $\boxed{-2}$

b) $h = \dot{y}t + \frac{1}{2}\ddot{g}t^2$

$h = v_0\sin\theta\left(\frac{v_0\sin\theta}{mg}\right) + \frac{1}{2}\cancel{mg}\left(\frac{v_0\sin\theta}{\cancel{mg}}\right)^2$

~~$h = \frac{(v_0\sin\theta)^2}{mg} + \frac{v_0\sin\theta^2}{2}$~~

$h =$

$* \leftarrow$ SIMPLIFY

$a = \frac{v_f^2 - v_i^2}{2h}$

$2ha = \cancel{v_f^2} - v_i^2$

(2) $\boxed{h = \frac{(v_i \sin\theta)^2}{2\cancel{m}g}}$

c) $\boxed{-v_0}$ ✓

d) $v_0\cos\theta \cdot t \cdot 2$

$\boxed{= \frac{2v_0^2\cos\theta\, v_0\sin\theta}{\cancel{m}g}}$

CORRECT CARRY FORWARD ERROR

~~CARRY FORWARD ERROR~~

10

Problem 2 (15 points)

A pendulum has length l (the string is "massless"). The bob has a mass of m. We release the bob with zero speed when the string makes an angle $\theta = 90°$ with the vertical. Friction of any kind can be ignored. The gravitational acceleration is g. Express your answers in terms of l, m, and g.

5 a. (5) What is the speed of the bob when it reaches its lowest point ($\theta = 0°$)?

0 b. (5) What is the tension in the string when $\theta = 0°$?

5 c. (5) How much work was done by gravity and how much by the tension in the string between the moment of release and the moment that the bob reaches its lowest point?

a) $h = l$ $\quad U = mgl$ $\quad \frac{1}{2}\cancel{m}v^2 = \cancel{m}gl$

$KE = \frac{1}{2}mv^2$ $\quad v^2 = 2gl$

$\boxed{v = \sqrt{2gl}}$ ✓

b) $\boxed{\text{Tension} = mg \uparrow}$ ✗

c) ~~work~~ $\boxed{W_G = mgl}$ ✓

$\boxed{W_T = 0}$ ← because $F_T \perp d$ at all points

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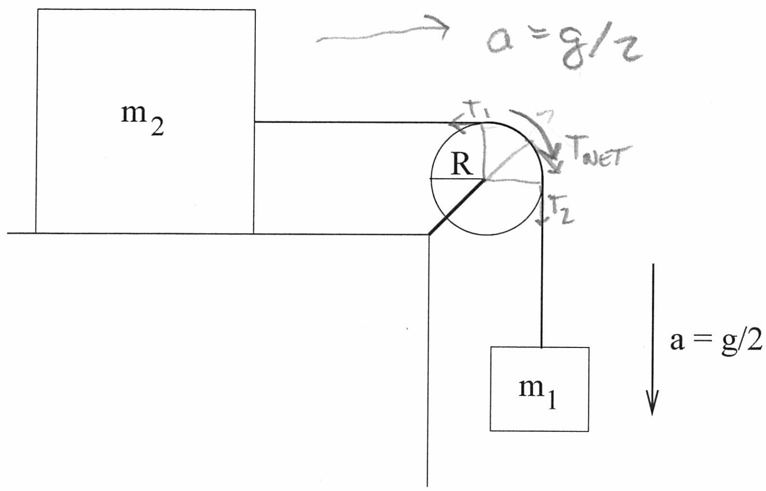
Problem 3 (24 points)

An unknown mass, m_1, hangs from a massless string and descends with an acceleration $g/2$. The other end is attached to a mass m_2 which slides on a frictionless horizontal table. The string goes over a uniform cylinder of mass $m_2/2$ and radius R (see figure). The cylinder rotates about a horizontal axis without friction and the string does not slip on the cylinder. Express your answers in parts b, c, and d in terms of g, m_2, and R.



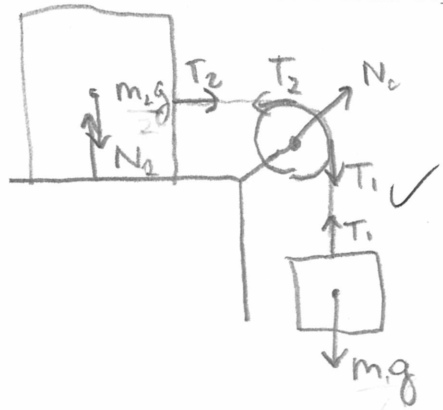
6 a. (6) Draw free-body diagrams for the cylinder and the two masses.

6 b. (6) What is the tension in the horizontal section of the string?

$\boxed{4}$ c. (6) What is the tension in the vertical section of the string?

6 d. (6) What is the value of the unknown mass m_1?

a)



b) $F = m_2 a$

$F = m_2 \cdot \frac{g}{2}$ ✓

$$\boxed{T_H = \frac{m_2 g}{2}}$$

c) $\tau_V = \tau_H = \tau_{NET}$ ✓

$\alpha R = \frac{g}{2}$ $\quad \alpha = \frac{g}{2R}$

$\alpha R = a$

$\tau_{NET} = I_0 \alpha$

$\tau_{NET} = I_0 \frac{g}{2R}$

$= MRg$

$\tau_V - \tau_H = MRg$ (−2)

$\tau_V - R\frac{mg}{2} = MRg$

$T_V \cancel{R} - \cancel{R}\frac{mg}{2} = M\cancel{R}g$

$T_V - \frac{mg}{2} = \frac{mg}{2}$

ALGEBRAIC ERROR $\boxed{-2}$

$\boxed{\frac{7}{8}}$

$$\boxed{T_v = mg}$$

d) $m_1 = ?$

CORRECT ALGEBRA & METHOD, CARRY FWD ERROR

$$F = ma$$

$$F_{net} = F_G - F_T$$

$$\frac{m_1 \cancel{g}}{2} = m_1 \cancel{g} - m_2 \cancel{g}$$

$$\frac{m_1 \cancel{g}}{2} = m_2 \cancel{g}$$

$$\boxed{m_1 = 2m_2}$$

CARRY FORWARD ERROR FROM (c)

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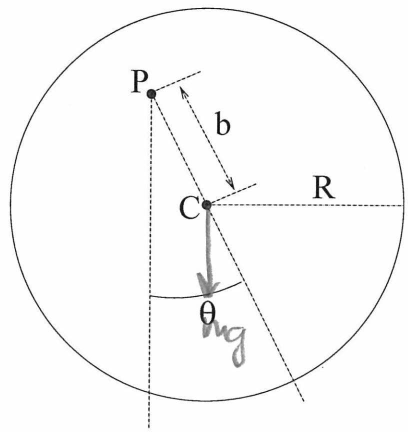
Problem 4 (20 points)

A solid, uniform disk of mass M and radius R is oscillating about an axis through P. The axis is perpendicular to the plane of the disk. Friction at P is negligibly small and can be ignored. The distance from P to the center, C, of the disk is b (see figure). The gravitational acceleration is g.



4 a. (4) When the displacement angle is θ, what then is the torque relative to point P?

4 b. (4) What is the moment of inertia for rotation about the axis through P?

2 c. (4) The torque causes an angular acceleration about the axis through P. Write down the equation of motion in terms of the angle θ and the angular acceleration.

As the disk oscillates, the maximum displacement angle, θ_{max}, is very small, and the motion is a near perfect simple harmonic oscillation.

4 d. (4) What is the period of oscillation?

4 e. (4) As the disk oscillates, is there any force that the axis at P exerts on the disk? Explain your answer.

a) $\boxed{\tau_P = mgb\sin\theta}$ ✓

b) $\boxed{I_P = \frac{1}{2}MR^2 + Mb^2}$ ✓

c) $\ddot{\theta} \leftrightarrow \theta$

$\frac{mgb\sin\theta}{I_P} = \ddot{\theta}$

$\ddot{\theta} - \frac{mgb\sin\theta}{\frac{1}{2}MR^2 + Mb^2} = 0$

$\boxed{\ddot{\theta} - \frac{2g\cancel{b}\sin\theta}{r^2\cancel{b}} = 0}$

ALGEBRA ERROR $\boxed{-2}$

d) $\theta = A\cos(\omega t + \varphi)$

$\dot{\theta} = -A\omega\sin(\omega t + \varphi)$

$\ddot{\theta} = -A\omega^2\cos(\omega t + \varphi)$

CARRY FWD ERROR

$\ddot{\theta} = -\omega^2\theta$

small angles assumption $\sin\theta = \theta$

$-\omega^2\theta - \frac{2gb\sin\theta}{r^2b^2} = 0$

$-\omega^2 = \frac{2gb\theta}{r^2b^2\theta}$ ✓

$\omega = \sqrt{\frac{2g}{r^2b}}$

$T = \frac{2\pi}{\omega}$ $\boxed{T = 2\pi\sqrt{\frac{r^2 b}{2g}}}$

e) Yes, a normal Force, otherwise the object would fall to the ground.

$\uparrow + mg$

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$V_{orb} = \sqrt{\frac{MG}{R}}$ $F = \frac{mMG}{R^2}$

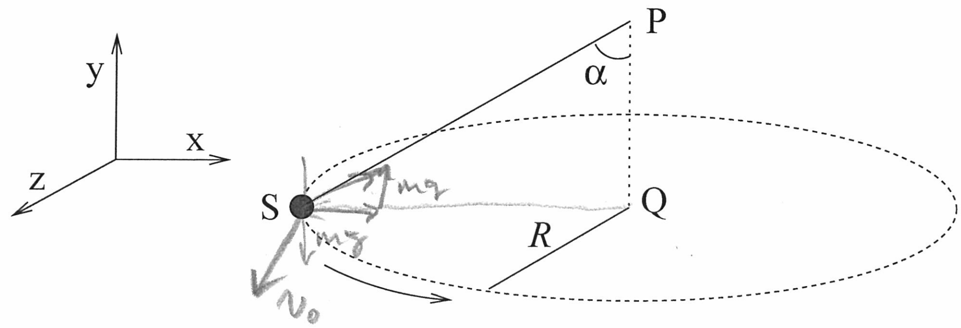
$V_{orb}^2 = \frac{MG}{R}$ $F = ma$

$V_{orb}^2 R = MG$ $a = \frac{MG}{R^2}$ $a = \frac{V_{orb}^2}{R}$

Problem 5 (25 points)

An apple of mass m is swung around on a string in a circle in a horizontal plane with a constant speed. The string makes an angle α with the vertical (see the figure). The radius of the circle is R; it takes τ sec for the apple to make one complete rotation; the direction of rotation is indicated in the figure; the apple, at S, is coming toward you. Q is the center of the circle. QP is vertical. SQ is in the +x-direction, QP in the +y-direction, and the +z-direction is tangent to the circle at S and points toward you. The gravitational acceleration is g; assume that the string is massless. Express your answers in terms of m, R, τ, and g.

τ and α?



4 a. (4) Make a free body diagram for the apple at S.

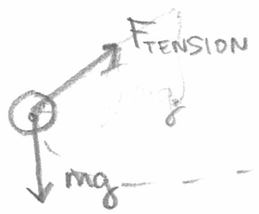
4 b. (4) What is the velocity of the apple at S (magnitude and direction), and what is its angular velocity ω?

4 c. (4) What is the apple's centripetal acceleration at S (magnitude and direction)?

2 d. (4) At S, what is the direction and magnitude of the sum of all forces acting on the apple. Indicate this *one* force in the figure or in a separate sketch, and mark this force unambiguously.

7 e. (9) What is α?

a)



d) F_{NET} P

$\boxed{-2}$ FORGOT TO PUT MAGNITUDE

b) $\omega = \frac{2\pi}{\tau}$ $\omega R = V \leftarrow$ at 90° relative to $\overline{SQ}$ (+z)

$\boxed{\omega = \frac{2\pi}{\tau}}$ $\boxed{V = \frac{2\pi R}{\tau}}$

c) Direction (+x)

$a_c = \frac{V^2}{R} = \frac{\left(\frac{2\pi R}{\tau}\right)^2}{R} = \frac{4\pi^2 R^{\cancel{2}}}{\tau^2 \cancel{R}}$ $\boxed{\frac{4\pi^2 R}{\tau^2}}$

e) $mg = \cos\alpha \, F_T$

$F_T \sin\alpha = ma$

$F_T = \frac{ma}{\sin\alpha}$

$mg = \frac{\cos\alpha \, \cancel{m} a}{\sin\alpha}$

$\cancel{m}g = \cot\alpha \, \cancel{m}a$

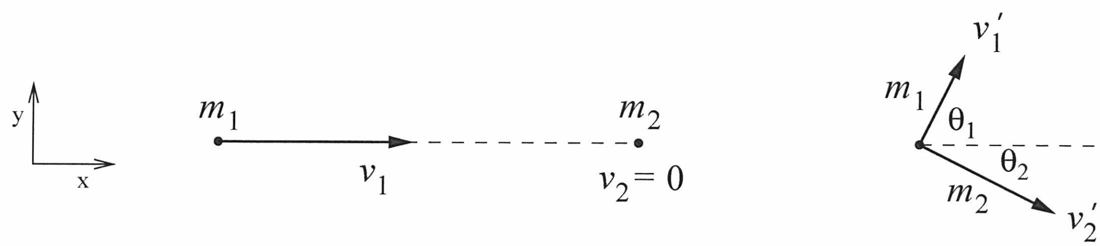
ALGEBRA ERROR $\boxed{-2}$

$\cot\alpha = \frac{g}{a}$

$\boxed{\cot\alpha = \frac{\tau^2 g}{4\pi^2 R}}$

$\boxed{18}$ $\boxed{\times \text{ DO}}$

Problem 6 (25 points)
A particle of mass m_1 and speed v_1 (in the +x-direction) collides with another particle of mass m_2. m_2 is at rest before the collision occurs, thus v_2=0. After the collision, the particles have velocities v_1' and v_2' in the x-y plane in the directions of θ_1 and θ_2 with the x-axis (see figure). There are no external forces. Express all your answers in terms of m_1, m_2, v_1, θ_1, and θ_2.



3 a. (3) What is the total momentum before the collision (direction and magnitude)?

4 b. (4) What is the total momentum after the collision (direction and magnitude)?

4 c. (4) What is the total kinetic energy before the collision?

3 d. (6) What is the ratio of the speeds v_2'/v_1'?

? $\boxed{4}$ e. (8) What is the magnitude (speed) of v_1'?

a) Total Momentum = $\boxed{m_1 v_1 \rightarrow}$ +x ✓

b) Total Momentum = $\boxed{m_1 v_1 \rightarrow}$ +x

c) $\boxed{\frac{1}{2} m_1 v_1^2}$ ✓

d) $\frac{v_1' \sin\theta_1}{v_1' \sin\theta_2} = \frac{-v_2' \cancel{\sin\theta_2}}{v_1' \cancel{\sin\theta_2}}$

$\boxed{\frac{v_2'}{v_1'} = \frac{m_1 \sin\theta_1}{m_2 \sin\theta_2}}$ $\boxed{-3}$ forgot masses

$m_1 v_1 = m_1 v_1' + \frac{v_1' m_2 \sin\theta}{\sin\theta_2}$

$m_1 v_1 = v_1'\left(m_1 + \frac{m_2 \sin\theta_1}{\sin\theta_2}\right)$

$\boxed{v_1' = \frac{m_1 v_1}{m_1 + \frac{m_2 \sin\theta_1}{\sin\theta_2}}}$ X

$\boxed{\text{forgot to divide by } \cos\theta_1}$

~~momentum~~ $\boxed{-4}$

e) $m_1 v_1 = m_1 v_1' + m_2 v_2'$ X ~~forgot~~

$\frac{v_2'}{v_1'} = \frac{\sin\theta_1}{\sin\theta_2}$ $v_2' = \frac{\sin\theta_1 v_1'}{\sin\theta_2}$ carry forward error

25

Problem 7 (25 points)

Imagine a spherical, non-rotating planet of mass M and radius R. The planet has no atmosphere. A spacecraft of mass m ($m \ll M$), is launched from the surface of the planet with speed v_0 at an angle of 30° to the local vertical. The rocket burn is very short. Thus you may assume that when the spacecraft has a speed v_0, it has not yet moved any appreciable distance.

4 a. (4) The speed v_0 is so high that the orbit is not bound. What is the minimum speed for which this is the case?

Now imagine that the orbit is bound and that in its subsequent orbit the spacecraft reaches a maximum distance of $15R$ from the center of the planet. At this distance the speed is V.

4 b. (4) What is the ratio of v_0/V?

4 c. (4) What is the total energy of the spacecraft immediately after launch?

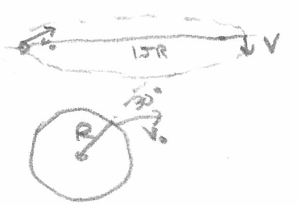
5 d. (5) What is the total energy of the spacecraft when it is farthest away from the planet?

8 e. (8) Write down one equation which would allow you to solve for v_0 in terms of M, G, and R (we are not asking you to solve this equation).

b)

a) $\boxed{V_{ESCAPE} = \sqrt{\frac{2MG}{R}}}$ ✓

b)



angular momentum is conserved about CM

$\vec{L}_1 = R v_0 \sin\theta$

$\vec{L}_2 = 15RV$

B) $\boxed{\frac{v_0}{V} = \frac{15}{\sin 30}}$ ✓

$\cancel{R} v_0 \sin\theta = 15\cancel{R}V$

$\boxed{v_0 = \frac{15V}{\sin 30}}$

c) $TE = \frac{-mMG}{2a}$

$a = 8R = \frac{15R+R}{2}$

$TE = \boxed{\frac{-mMG}{16R}}$ ✓

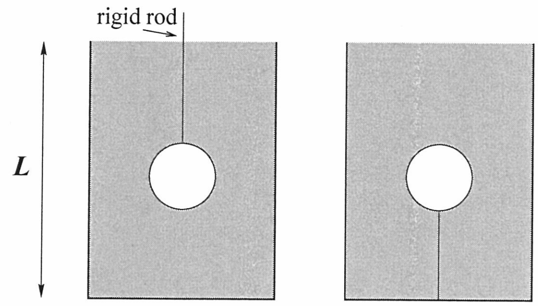
d) $TE = \text{Constant} = \boxed{\frac{-mMG}{16R}}$ ✓

e) $\boxed{\frac{1}{2} m v_0^2 - \frac{mMG}{R} = -\frac{mMG}{16R}}$

(18)

Problem 8 (18 points)

A cylindrical container of length L is full to the brim with a liquid which has mass density ρ. It is placed on a weigh-scale; the scale reading is W. A light ball which would float on the liquid if allowed to do so, of volume V and mass m is pushed gently down and held beneath the surface of the liquid with a rigid rod of negligible volume as shown on the left.



4 a. (4) What is the mass M of liquid which overflowed while the ball was being pushed into the liquid?

6 b. (6) What is the reading of the scale when the ball is fully immersed? Give your reasons.

8 c. (8) If instead of being pushed down by a rod, the ball is held in place by a thin string attached to the bottom of the container as shown on the right. What is the tension T in the string, and what is the reading on the scale?

a) $\boxed{M_{displaced} = \rho V}$ ✓

b) ~~Th~~ $\boxed{W}$, ✓ because the rod exerts a downward force equal to that of the displaced water's weight

c) i) $\boxed{T = \rho V g - mg}$ ✓

ii) $\boxed{W^* = W - \rho V g - mg}$

$\boxed{0}$

Problem 9 (15 points)

Show that if the temperature T in the atmosphere were independent of altitude, the pressure p as a function of altitude h would be

$$p = p_0 e^{-\frac{mgh}{kT}}$$

where m is the average mass of an air molecule, and p_0 is the pressure at sea level.

$$P(h) = P_0 e^{-\frac{mgh}{kT}}$$

$$\frac{dP}{dy} = -\rho g$$

$$P_0 V = NkT$$

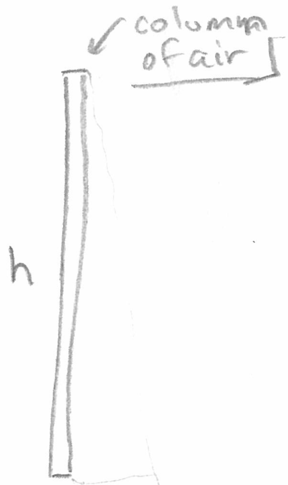
$$P_0 V_0 = NkT$$

$$\rho = \frac{M}{V}$$

$$\rho = Nm$$

$$P_0 = \frac{N}{V} kT$$

$$\underline{P(h) = \rho g h}$$



$$P_0 = \frac{\rho}{m} kT \qquad P(h) = \rho g h$$

$$P(h) = \int \rho g h = \boxed{\tfrac{1}{2}\rho g h}^2$$

$$\cancel{\tfrac{1}{2}\rho g h^2 = P(h)}$$

$$\frac{Nm}{V} = \rho$$

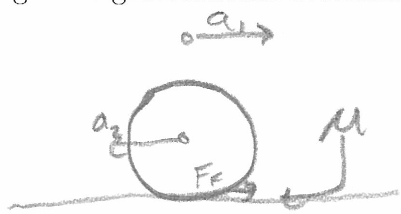
$$\frac{N}{V} = \frac{\rho}{m}$$

$$\frac{\text{\# of balls}}{\text{Volume}} \qquad \text{density } \frac{\text{weight of ball} \cdot \text{\# of balls}}{\text{Volume}}$$

4

Problem 10 (17 points)

A bowling ball of mass m and radius R sits on the smooth floor of a subway car. If the car has a horizontal acceleration a_1, what is the acceleration a_2 of the ball? Assume that the ball rolls without slipping. The gravitational acceleration is g.



$$\tau_c = Rmg\mu$$

$$\underbrace{\alpha = Ra}_{\text{CONDITION OF PURE ROLL}}$$

$$\tau_c = I_o\alpha$$

$$\cancel{R}\cancel{m}g\mu = \frac{2}{5}\cancel{M}R^{\cancel{2}}\alpha$$

$$g\mu = \frac{2}{5}R\alpha$$

$$g\mu = \frac{2}{5}R^2 a$$

$$\boxed{a_2 = \frac{5g\mu}{2R^2}}$$