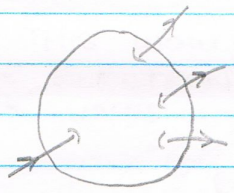
UNDERSTANDING GAUSS' LAW

GAUSS' LAW STATES THAT THE FLUX POURING OUT OF A CLOSED SURFACE IS EQUAL TO THE NUMBER OF CHARGES (NET) WITHIN ITS VOLUME (DIVIDED BY PERMITIVITY CONSTANT):

$$\oint \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon_0}$$

ANOTHER WAY OF LOOKING AT THIS IS THAT THE FLUX OF A CLOSED SURFACE IS THE SAME AS THE DIVERGENCE WITHIN THE SURFACE

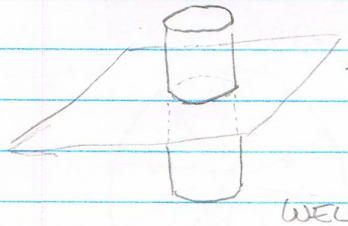
$$\oint = \iiint \text{div}(F)\, dV$$ ← NOW REMEMBER DIVERGENCE IS THE SOURCE RATE PER UNIT VOLUME, MEANING THIS EXPRESSION IS EQUIVALENT TO NET SOURCES/SINKS OF FLUX.



SUPPOSE THIS IS A CLOSED SURFACE. WE SEE THAT 3 LINES ARE EXITING AND ONE IS ENTERING, GIVING A NET FLUX OF 2 LINES OUT (NORMALIZED SOMEHOW)

THIS IMPLIES THERE MUST EXIST A POSITIVE CHARGE WITHIN THE SURFACE.

GAUSS' LAW ALLOWS YOU TO CALCULATE CHARGE FROM FLUX OR VICE VERSA, IF ONE IS UNKNOWN OR HARD TO CALCULATE.

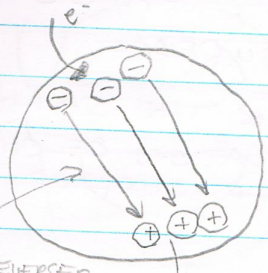


← HERE WE HAVE A CHARGED SHEET WITH A CHARGE DENSITY OF σ, WHAT IS THE FIELD ABOVE?

WELL, IF THE PLATE IS INFINITE, FLUX MUST ONLY BE GOING UP OR DOWN, SINCE IT ISN'T FLOWING OUT OF A TANGENTIAL SURFACE.

THEREFORE THE FLUX MUST BE COMING OUT OF THE TOPS, WHICH HAVE AREA = 1 EACH. IF THE CHARGE IS σ, THEN THE FORCE ON EACH

UNDERSTANDING CHARGES IN CONDUCTORS

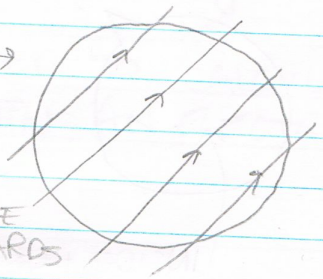


NOTE THESE ARROWS ARE REVERSED FROM THE DIRECTION OF CURRENT WHICH IS ⊕ → ⊖

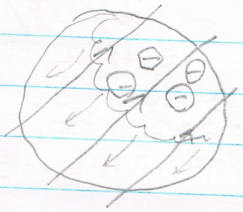
← IN A CONDUCTOR CHARGES MOVE AROUND FREELY (ELECTRON GAS) THEREFORE IF WE PLACED EXCESS ELECTRONS ON ONE SIDE AND REMOVED ELECTRONS ON THE OPPOSITE SIDE, THERE WOULD BE A CURRENT TO IMMEDIATELY RESOLVE THE IMBALANCE

THIS IMPLIES THAT, AT REST, A CONDUCTOR CAN NEVER HAVE ELECTRIC FIELDS RUNNING THROUGH IT. OTHERWISE A CURRENT WOULD BE CREATED AS THE ELECTRON GAS 'FALLS' INTO THE FIELD.

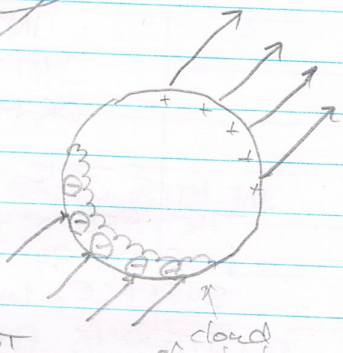
IMAGINE A HYPOTHETICAL CONDUCTOR WHICH DOES HAVE INTERNAL E-FIELD LINES →



THIS WOULD IMMEDIATELY CAUSE THE ELECTRON GAS TO 'FALL' TOWARDS THE LINES



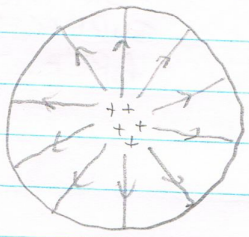
THIS 'FALL' OF THE GAS WOULD PRODUCE A CHARGE INEQUALITY, ACCUMULATING UNTIL THE INTERNAL FIELD LINES WERE NEUTRALIZED



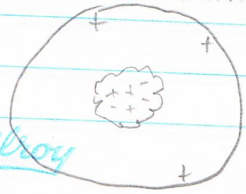
cloud of electron gas is shrunken to the surface from 'falling'

THIS ALSO MEANS CONDUCTORS CANNOT HAVE INTERIOR CHARGE.

IMAGINE A CONDUCTOR WHICH HAS BEEN ADDED INTERIOR CHARGE

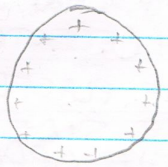


IMMEDIATELY THE ELECTRON GAS COLLAPSES ON THE POSITIVE CHARGE, LEAVING POSITIVE CHARGE ONLY ON THE SURFACE

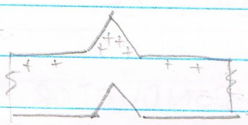


Hilroy

OVER A SYMMETRICAL OBJECT, CHARGE IS PUSHED EVENLY OVER THE SURFACE

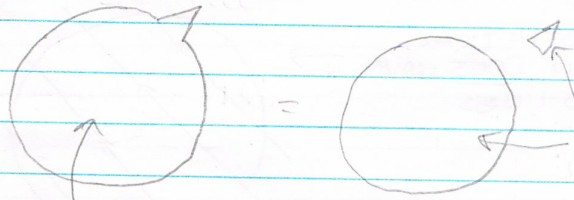


THIS IS ALSO FOR THE SAME REASON. AN ACCUMULATION OF CHARGE ON ONE PART OF THE SURFACE WOULD SPREAD OUTWARD.



IN A NON-SYMMETRICAL SURFACE, CHARGE CONCENTRATES ON CORNERS AND POINTS.

TO UNDERSTAND WHY, REALIZE THAT THE ELECTRIC POTENTIAL (LIKE GRAVITATIONAL POTENTIAL) CANNOT HAVE A MIN OR MAX IN A CHARGELESS REGION

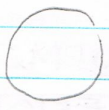


* THESE 2 REGIONS
** MUST BE AT EQUIPOTENTIALS.

INSIDE HERE THERE IS NO CHARGE ∴ THERE IS NO MIN OR MAX POTENTIAL

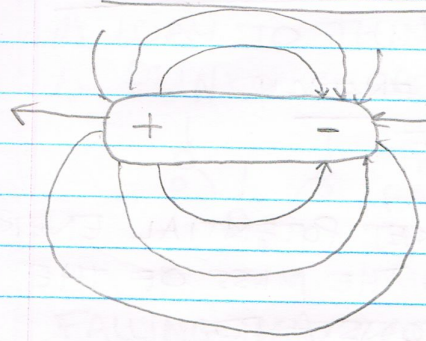
THE POTENTIAL WITHIN IS CALCULATED BY THE AVERAGE OF SURFACE POTENTIAL, BUT

IF THIS WEREN'T THE CASE THE REGION B, BEING ENCLOSED MOSTLY BY THE TRIANGULAR SURFACE, WOULD HAVE A LOWER INTERNAL POTENTIAL THAN A. THIS MEANS THE ELECTRON GAS WOULD FALL INTO A UNTIL THE POTENTIAL DROP WAS NEUTRALIZED.



A HAS MORE AREA THAN B, SO THE SURFACE CHARGE ON B MUST BE LARGER, TO ACCOUNT FOR THE SAME SURFACE-TO-INTERIOR AREA.

UNDERSTANDING DIPOLES

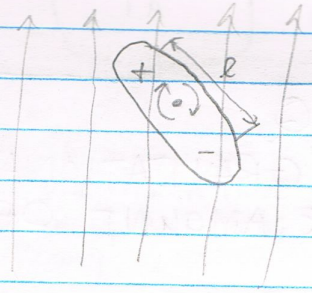


A DIPOLE IS AN OBJECT (SUCH AS AN ATOM) WHICH HAS A POSITIVE AND NEGATIVE END.

ONE ATTRIBUTE IS CALLED THE DIPOLE MOMENT, WHICH IS $p = Q\ell$

- Q: charge on one side
- ℓ: length separating the charges.

THIS MOMENT CAN BE USED TO CALCULATE TORQUE OF A DIPOLE WITHIN AN ELECTRIC FIELD. FOR EXAMPLE:



$$\tau = p \times E$$

TORQUE IS THE MOMENT CROSSED WITH THE IMMERSED ELECTRIC FIELD

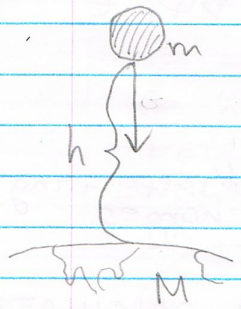
WHEN DIPOLES ARE ATOMS, THEY WILL ALIGN AND UNALIGN IN THE PRESENCE OF THE FIELD (DUE TO SHO AND THERMAL FLUCTUATIONS). IF THEY COLLIDE WHILE SPINNING, HEAT IS CREATED (E.G. A MICROWAVE)

Hilroy

UNDERSTANDING VOLTAGE AND ELECTRIC POTENTIAL

$$\text{ELECTROSTATIC POTENTIAL} = \frac{\text{POTENTIAL ENERGY}}{\text{CHARGE}}$$

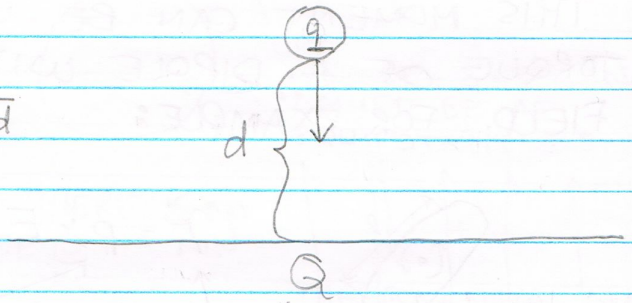
THINK OF ELECTROSTATIC LIKE POTENTIAL ENERGY WITH GRAVITY, BUT WITH THE MASS OF THE FALLING OBJECT HELD CONSTANT.



WITH GRAVITY, POTENTIAL ENERGY IS EQUAL TO $U = -\frac{mMG}{h}$ (NOTE IT IS A NEGATIVE VALUE)

NOW THE ANALOGY TO VOLTAGE IS SIMILAR:

$U = \frac{-qQ}{4\pi\varepsilon_0 d}$ (NEGATIVE FOR ATTRACTION, POSITIVE FOR REPULSION)

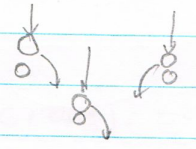


NOW IMAGINE WE HAVE A "GRAVITATIONAL VOLTAGE" IT WOULD SIMPLY BE THE AMOUNT OF ENERGY PER KILOGRAM.

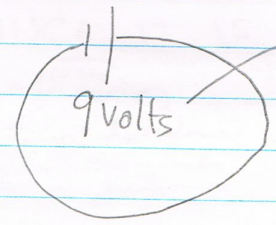
VOLTAGE IS THE DIFFERENCE BETWEEN TWO ES-POTENTIALS. SO IF YOU HAVE A 12-VOLT CURRENT, THAT MEANS EACH UNIT OF CHARGE IS 'FALLING' WITH AN ENERGY OF 12 J.

A CURRENT IS SIMPLY AN ONGOING WAVE OF THESE 'FALLING' CHARGES, WHERE THE VOLTAGE IS AN INDICATOR OF HOW MUCH ENERGY THEY HAVE

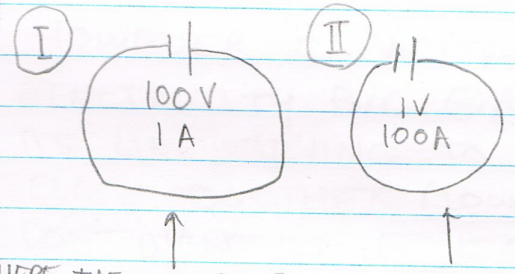
A WAY TO THINK OF THIS IS LIKE IMAGINING A PLINKO CURRENT, LOOPING AROUND



THE COLLISIONS KEEP THE PLINKO BALLS FROM ACCELERATING, SO THE "GRAVITATIONAL VOLTAGE" IS A MEASURE OF HOW HARD THEY ARE FALLING INTO EACH PEG, AND THE ENERGY RELEASED WHEN THEY DO. (HERE, YOU CAN INCREASE THE ENERGY, BY INCREASING M, OR THE PULLING GRAVITY)



THIS 9 VOLTS MEANS THAT EACH COLOUMB IS 'FALLING' FROM A POTENTIAL ENERGY GAP OF 9 JOULES



HERE THE ELECTRONS ARE 'FALLING' A HUGE DISTANCE, BUT GOING SLOWLY

HERE THEY ARE FALLING ONLY A SHORT GAP, BUT DOING IT MUCH FASTER

HOW CAN THESE TWO SITUATIONS COEXIST?

WELL, OHM'S LAW STATES THAT CURRENT IS PROPORTIONAL TO VOLTAGE: $I = \frac{V}{R}$ OR $V = IR$

SO (I) CAN EXIST IF THERE IS A LOT OF RESISTANCE (LARGE FALL, THROUGH WATER) AND (II) CAN EXIST IF THERE IS LOW RESISTANCE (SMALL FALL, THROUGH A VACUUM)

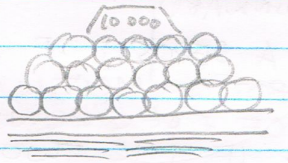
Hilroy

UNDERSTANDING CAPACITORS

$$\text{CAPACITANCE} = \frac{Q}{V} = \frac{\text{CHARGE}}{\text{VOLTAGE}}$$

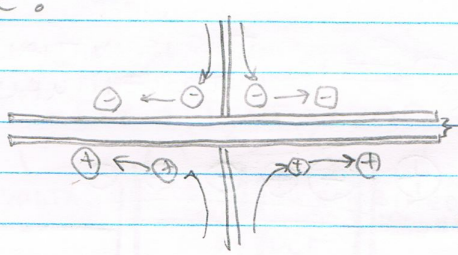
CAPACITANCE IS HIGH WHEN A LOT OF CHARGE CAN BE STORED AT A LOW ENERGY LEVEL. THINK OF IT LIKE STORING A LOT OF MASS, BUT LOW TO THE GROUND.

THE GRAVITATIONAL ANALOGY ISN'T AS ILLUMINATING SINCE IT IS ALWAYS ATTRACTIVE. AFTER ALL, IMAGINING A MACHINE WHICH JUST KEEPS A LOT OF WEIGHT CLOSE TO THE EARTH ISN'T IMPRESSIVE.



HOWEVER, CAPACITANCE IS INTERESTING IN ELECTRICITY B/C ⊖/⊖ & ⊕/⊕ REPEL. SO ITS LIKE TRYING TO PUT A LOT OF THINGS (HIGH CHARGE) CLOSE TOGETHER (LOW VOLTAGE) BUT THE THINGS DON'T WANT TO STICK TOGETHER.

A CAPACITOR IS A TOOL DESIGNED TO MINIMIZE THE VOLTAGE ("HEIGHT") WHILE MAXIMIZING THE CHARGE:

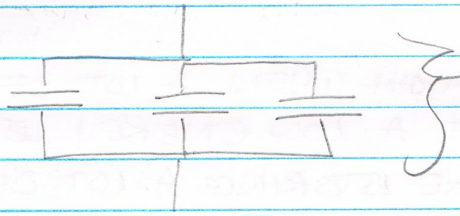


HERE THE GAP IS SMALL, SO THE POTENTIAL ENERGY (IF THE TWO PLATES TOUCHED) IS ALSO SMALL.

BUT THE SURFACE AREA OF THE PLATES IS LARGE, SO A LOT OF CHARGE CAN BE PUSHED ONTO IT

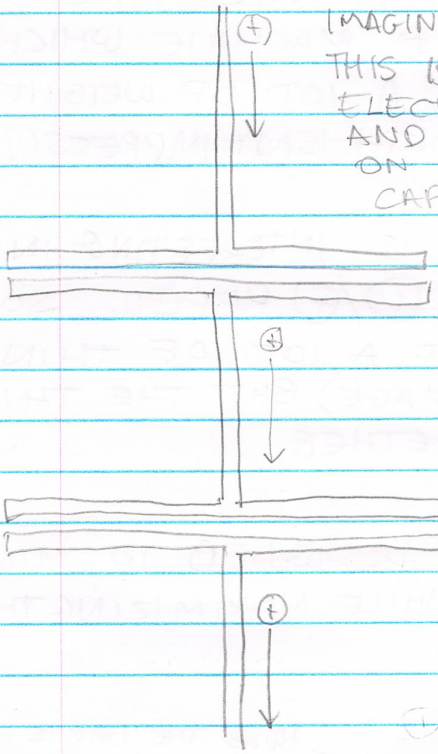
Hilroy

WHEN CAPACITORS ARE CONNECTED IN PARALLEL, THEIR CAPACITY IS ADDED



think of this as 3 storage containers, each filling with charge

WHEN CAPACITORS ARE IN SERIES, THE CAPACITY IS LOWER

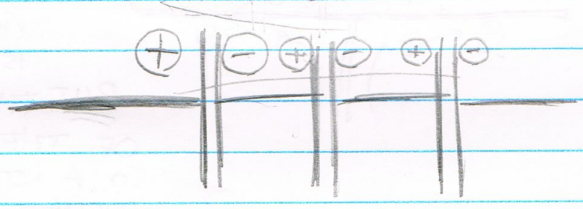


IMAGINE ⊕ FLOWING DOWNWARD

THIS IS LIKE HAVING A DEPOSIT OF ELECTRONS ON THE BOTTOM-MOST PLATE AND A DRAINING OF ELECTRONS ON THE TOPMOST. IF THE TWO CAPACITORS WERE IDENTICAL, THAT WOULD FIRST YIELD THE CAPACITY OF ONE.

SECONDLY, THIS INDUCES A POSITIVE CHARGE HERE, PUSHING ELECTRONS UP TO THE TOP HALF, HOWEVER, EACH WAVE DIMINISHES THE TOTAL CAPACITY AS THE PULL GETS WEAKER.

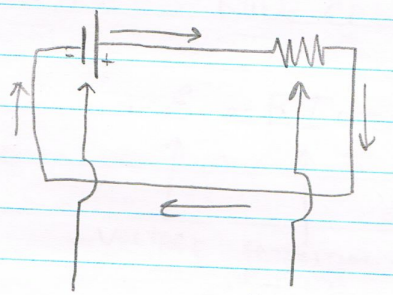
THIS IS INITIAL STATE, BUT IT EVENTUALLY EVENS OUT.



UNDERSTANDING KIRCHOFF'S RULE*

KIRCHOFF'S RULE TELLS US THAT THE SUM OF ALL EMFS AND POTENTIAL DROPS MUST EQUAL ZERO

*NOTE, FIRST, WHETHER MAGNETIC INDUCTION WILL PLAY A ROLE IN THE CIRCUIT. IF IT DOES, THIS RULE WON'T WORK!



A SIMPLE EXAMPLE OF A 1-LOOP CIRCUIT.

IMAGINE THE BATTERY LIKE A PUMP, PUSHING CURRENT THROUGH THE RESISTOR.

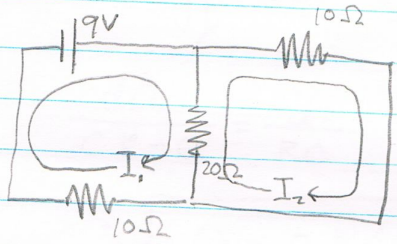
HERE THERE IS AN EMF OF $\mathcal{E}$

HERE, THE POTENTIAL DROP MUST EQUAL $\Delta V = -IR$

$I = \frac{\mathcal{E}}{R}$ THE CURRENT MUST THEREFORE BE THE RATIO BETWEEN EMF AND RESISTANCE.

CALLED VOLTAGE WHICH IS = DIFFERENCE IN ELECTROSTATIC POTENTIAL

NOW WHAT ABOUT A MORE COMPLICATED EXAMPLE?



KIRCHOFF'S RULE STATES THAT, FOR EACH CURRENT ALL $\mathcal{E}$ + potential drops EQUALS ZERO

FLOW THROUGH BOTTOM

$$-9V + (I_1 - I_2)\cdot 20\Omega + I_1 \cdot 10\Omega = 0$$
$$-0V + (I_2 - I_1)\cdot 20\Omega + I_2 \cdot 10\Omega = 0$$

VOLTAGE — FLOW OF COMBINED CURRENTS THROUGH MIDDLE — FLOW THROUGH TOP

$$-9 + 20(I_1 + I_2) + 10I_1 = 20(I_2 - I_1) + 10I_2$$

$$-9 + 10I_1 = 10I_2$$

$$I_2 = \frac{-9 + 10I_1}{10}$$

$$\left(I_1 - I_1 - \frac{9}{10}\right)20 + 10\left(\frac{-9 + 10I_1}{10}\right) = 0$$

$$-18 - 9 + 10I_1 = 0$$

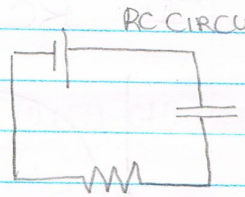
$I_1 = 2.7\,A$

$I_2 = 1.6\,A$ Hilroy

KEEP IN MIND WITH KIRCHOFF'S RULE THAT THERE MAY BE MULTIPLE VOLTAGE SOURCES, RESISTORS OR CAPACITORS (CAN'T USE WITH SELF-INDUCTION COILS!)

UNDERSTANDING RC CIRCUITS

AN RC CIRCUIT HAS A RESISTOR AND A CAPACITOR IN SOME COMBINATION



KIRCHOFF'S RULE CAN ALSO WORK HERE:*

*ONLY IF THE CHANGE IN CURRENT DOES NOT CREATE A STRONG ENOUGH MAGNETIC FIELD TO REMOVE CONSERVANCY OF FORCE

$$\mathcal{E} - RI - \frac{Q}{C} = 0$$

- $\mathcal{E}$: VOLTAGE
- RI: POTENTIAL DROP DUE TO RESISTANCE
- $\frac{Q}{C}$: $V = \frac{Q}{C}$ FOR CAPACITORS

AT FIRST GLANCE THE ABOVE EQUATION APPEARS UNSOLVABLE, BUT REMEMBER CURRENT IS CHANGE IN CHARGE PER UNIT TIME:

$$\mathcal{E} - R\frac{dQ}{dt} - \frac{Q}{C} = 0$$

← NOW THIS IS IN THE FORM OF A DIFFERENTIAL EQUATION

$$\Downarrow$$

$$\frac{dQ}{C\mathcal{E} - Q} = \frac{dt}{RC} \Rightarrow$$

INTEGRATE BOTH SIDES TO GET

$$-\ln(C\mathcal{E} - Q) + \ln(C\mathcal{E}) = \frac{t}{RC}$$

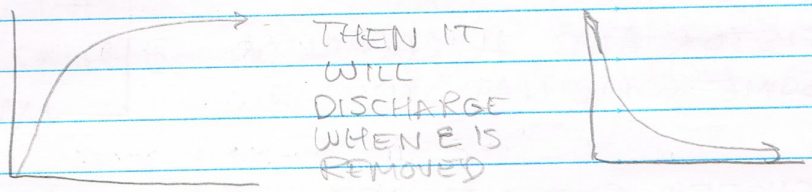
$$\therefore \boxed{\begin{aligned} Q &= C\mathcal{E}\left(1 - e^{-t/RC}\right) \\ \frac{dQ}{dt} &= I = \frac{\mathcal{E}}{R}\left(e^{-t/RC}\right) \end{aligned}}$$

← amount of charge on capacitor as a function of time

REMEMBER THESE FORMULAE, B/C YOU DON'T KNOW HOW TO DO DIFF. EQUATIONS YET!

Hilroy

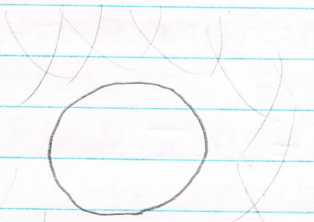
AN RC CIRCUIT WILL CHARGE UP



RC IS CALLED THE "TIME CONSTANT" OF THE CIRCUIT OR "RELAXATION TIME"

AT $t = RC$, CURRENT IS AT ~37% OF MAX.

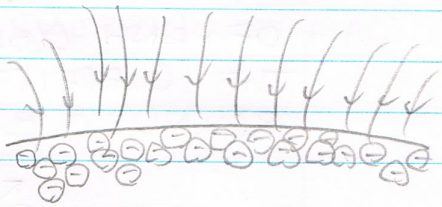
UNDERSTANDING ELECTRIC BREAKDOWN & CORONA DISCHARGE



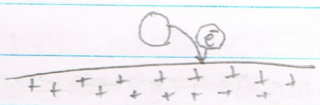
IMAGINE AN OBJECT WITH CHARGE IMMERSED IN AIR.

THE AIR DOESN'T CONDUCT SO THE CHARGE REMAINS ON THE OBJECT.

AS THE VOLTAGE INCREASES, MORE AND MORE CHARGE IS PACKED ONTO THE SURFACE OF THE CONDUCTOR →



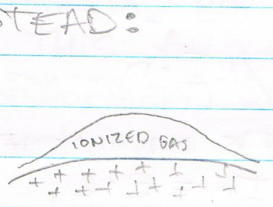
EVENTUALLY, THE CHARGE MAY BE HIGH ENOUGH TO IONIZE A MOLECULE OF NEARBY GAS.



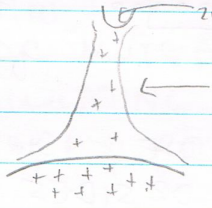
THIS IONIZES THE AIR, RESULTING IN THE GAS MOLECULE TO DEPOSIT THE EXTRA CHARGE ELSEWHERE.

THIS PROCESS IS CALLED CORONA DISCHARGE. OR THE LEAKING OF CHARGE OFF THE OBJECT.

IF THE VOLTAGE IS TOO HIGH, IT WILL CAUSE ELECTRIC BREAKDOWN INSTEAD:



→



← THIS CREATES A CONDUCTING CHANNEL, UNLOADING THE EXCESS CHARGE IMMEDIATELY, RESULTING IN A SPARK.

THE ELECTRIC BREAKDOWN POINT IN AIR IS PROPORTIONAL TO THE R^2 ← SURFACE POTENTIAL OF THE SPHERE, BUT CAN BE EXACERBATED BY SMALL REGIONS OF CONCENTRATED CHARGE DUE TO POINTS OR EDGES.

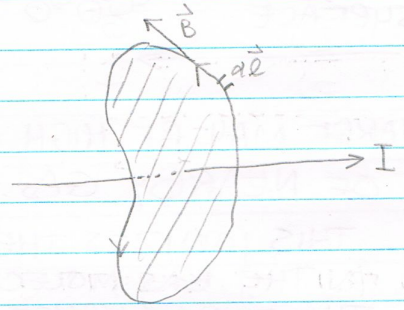
Hilroy

UNDERSTANDING AMPERE'S LAW

AMPERE'S LAW IS AN EASIER WAY TO CALCULATE MAGNETIC FIELDS THAN BIOT-SAVART.

THE LAW SAYS: $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$

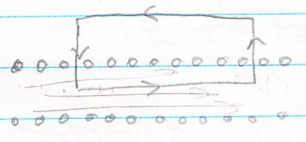
PUT INTO OTHER WORDS, THE LINE INTEGRAL OF ANY CLOSED LOOP IS PROPORTIONAL TO THE CURRENT FLOWING THROUGH ANY SURFACE BOUNDED BY THAT LOOP



REMEMBER THAT THE LINE INTEGRAL IN SPACE AROUND A VECTOR FIELD IS THE SAME AS THE CURL • NORMAL VECTOR OF THE SURFACE AREA!

THIS MEANS THAT THE MAGNETIC FIELD IS RELATED TO THE CURL OF BIOT SAVART'S LAW.

THIS GIVES US A DERIVATION FOR A SOLENOID



HOW MUCH MAGNETIC FIELD IS WITHIN THIS SOLENOID?

WELL $\mu_0 I \cdot n_{\text{WINDINGS}} = \oint \vec{B} \cdot d\vec{\ell}$

HOWEVER, WE KNOW THERE IS NO MAGNETIC FIELD IN THE TOP, OR SIDES, $\therefore$ $\underbrace{\mu_0 I n}_{\substack{\text{CURRENT} \times \\ \#\text{WINDINGS} \\ \times \text{PERMEABILITY} \\ \text{CONSTANT}}} = \underbrace{B \cdot \ell}_{\substack{\text{FIELD TIMES} \\ \text{LENGTH}}}$

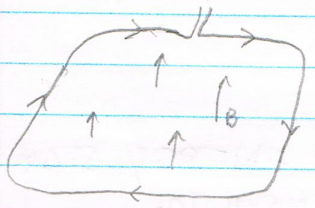
IN OTHER WORDS THE MAGNETIC FIELD STRENGTH IS EQUAL TO THE CURRENT × WINDINGS DIVIDED BY LENGTH (ℓ/μ_0)

ELECTROMAGNETIC INDUCTION

WHEN ELECTRIC FIELDS MOVE, IT CREATES A MAGNETIC FIELD. SIMILARLY, A CHANGING MAGNETIC FIELD INDUCES AN EXTRA ELECTRIC FIELD WHICH RESISTS THE CHANGE IN THE FIELD. THINK OF THIS LIKE MAGNETIC "FRICTION"

INDUCED EMF $\mathcal{E}_{ind}$ = RATE OF SWEEPING OF MAGNETIC FLUX.

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$



SUPPOSE THAT A UNIFORM MAGNETIC FIELD IS SUDDENLY TURNED ON. THIS WILL INDUCE AN EMF ON THE COIL SINCE $\frac{d\Phi_B}{dt} = \frac{dB}{dt} \cdot \text{Area}$

THIS INCREASE IN FLUX UP, CREATES A CCW FLOW OF CURRENT.

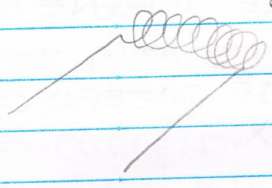
THIS INDUCED CURRENT IS NOT CONSERVATIVE (THINK FRICTION) ∴ KIRCHOFF WONT WORK AND TOTAL VOLTAGE - POTENTIAL DROPS WILL NOT BE ZERO

THIS ALLOWS FOR MUTUAL INDUCTANCE (TRANSFORMERS) OR TRANSLATING ROTATIONAL MECHANICAL MOTION INTO ELECTRICITY (DYNAMOS)

Kilroy

UNDERSTANDING SELF-INDUCTANCE

SELF INDUCTANCE IS THE TENDENCY OF A CIRCUIT, OPERATING IN A TIME-DEPENDENT CURRENT, TO RESIST CHANGES TO THAT CURRENT



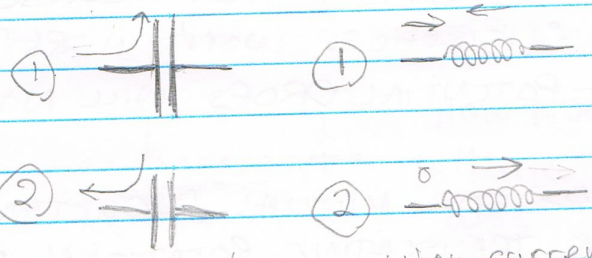
← as current changes, magnetic flux changes, slowing current change. As current stops, the abrupt decrease in magnetic flux generates a current.

L = Constant of self-inductance

$$\mathcal{E} = -L\frac{dI}{dt}$$

$\frac{1}{2}LI^2$ ← MAGNETIC ENERGY STORED IN SELF INDUCTOR

THE DIFFERENCE BETWEEN A CAPACITOR AND SELF-INDUCTOR IS THAT THE CREATE OPPOSITE PUSHES OF CURRENT



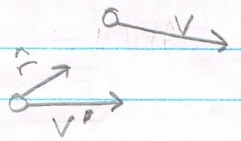
when current stops, charge falls back

when current stops, charge pushes forward.

MAGNETISM

THE MAGNETIC FORCE GENERATED BY A MOVING POINT CHARGE =

$$\frac{\mu_0}{4\pi} \cdot \frac{qq'}{r^2} \cdot v \times (v' \times \hat{r})$$



THIS EQUATION STATES THAT THE MAGNETIC FORCE IS LINEARLY PROPORTIONAL WITH THE PRODUCT OF THE CHARGES DIVIDED BY DISTANCE SQUARED AND THE CROSS PRODUCT OF THE TWO VELOCITIES (AND THE UNIT DISTANCE VECTOR)

related also to speeds combined

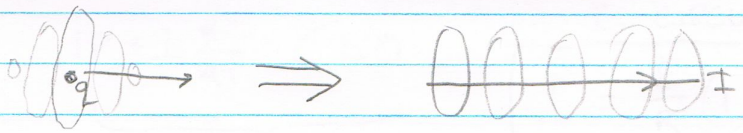
$$\frac{qq'}{r^2} \cdot v \times (v' \times \hat{r})$$

basic relationship between distance and charge strength

the velocity vector is rotated 90° so it points toward the other charge when parallel

BIOT-SAVART'S LAW

THIS IS AN EXTENSION OF THE UNDERSTANDING FOR POINT CHARGES. INSTEAD OF A SINGLE CHARGE, A CONTINUOUS CURRENT OF CHARGES GENERATES A STATIC MAGNETIC FIELD



THE TOTAL MAGNETIC FIELD CAN BE CALCULATED BY

$$B = \int \frac{\mu_0}{4\pi} \cdot \frac{I\, d\vec{\ell} \times \vec{r}}{r^2}$$

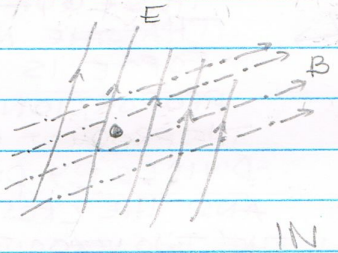
NOTE THIS IS SIMPLY AN INTEGRATION OF ALL THE POINT MAGNETIC FIELDS CREATED AT EACH SPOT

Hilroy

UNDERSTANDING LORENTZ FORCE

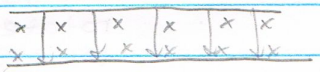
LORENTZ FORCE OCCURS WHENEVER BOTH A MAGNETIC AND ELECTRIC FORCE ACT ON A PARTICLE

FOR EXAMPLE:



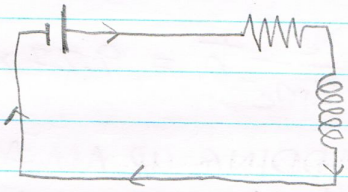
$$F = qE + qv \times B$$

THIS IS PARTICULARLY RELEVANT IN A VELOCITY SELECTOR WHERE THE FIELDS ARE ALIGNED AT 90°, SO THAT ONLY PARTICLES WHOSE $qv \times B$ TERM PERFECTLY OPPOSES THE qE TERM WILL CONTINUE STRAIGHT AHEAD



UNDERSTANDING RL CIRCUITS

AN RL-CIRCUIT IS MADE BY COMBINING A RESISTOR WITH A SELF-INDUCTOR.



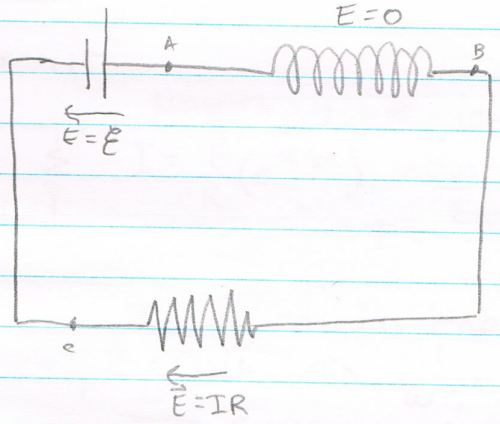
HERE KIRCHOFF'S RULE WON'T WORK BECAUSE WE ARE IN THE PRESENCE OF A NONCONSERVATIVE FORCE.

HOWEVER FARADAY'S LAW WORKS FINE:

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

TOTAL EMF-POTENTIAL DROPS IS EQUAL TO THE CHANGE IN MAGNETIC FLUX (MINUS)

TRANSLATED TO SELF-INDUCTANCE CONSTANT



GO FROM POINT TO POINT ALONG THE CIRCUIT AND ADD THE EMFS

$$AB + BC + CA = \frac{d\Phi_B}{dt}$$

$$0 + IR - \mathcal{E} = -L\frac{dI}{dt}$$

NOW WE CAN DERIVE EQUATIONS JUST LIKE WITH RC CIRCUITS

$$IR + L\frac{dI}{dt} - \mathcal{E} = 0$$

ANOTHER DIFFERENTIAL EQUATION, EXCEPT THIS TIME THE

$$\frac{Q}{C} \longrightarrow L\frac{d^2Q}{dt^2}$$

2ND ORDER DERIVATIVE

$$\boxed{I = \frac{\mathcal{E}}{R}\left(1 - e^{-tR/L}\right)}$$

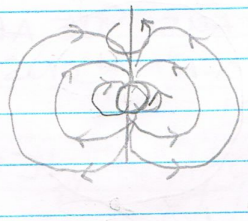
REMEMBER THIS BECAUSE IT USES DIFFERENTIAL EQUATIONS WHICH YOU CAN'T DO YET.

Hilroy

UNDERSTANDING A BOHR MAGNETON

A BOHR MAGNETON IS THE MAGNETIC DIPOLE CREATED BY AN ELECTRON SPINNING.

$$\mu_{SPIN} = \frac{e}{2m_e}\hbar = 9.27 \times 10^{-24}$$



WHEN ADDING UP AN ATOM'S VALUES FOR ELECTRON SPIN AND ORBITAL MAGNETIC MOMENTS, SOME ATOMS HAVE A DIPOLE MOMENT WHICH IS ALWAYS A MULTIPLE OF THE VALUE ABOVE.

UNDERSTANDING AC-POWER

AC POWER ALTERNATES THE INTENSITY AND DIRECTION OF CURRENT BY:

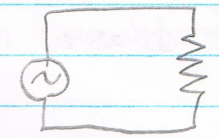
$$\mathcal{E} = \mathcal{E}_{max} \sin(\omega t)$$

this is higher than the average

115-AC has an $\mathcal{E}_{max}$ of $115 \cdot \sqrt{2}$ but delivers the same average power as 115 DC

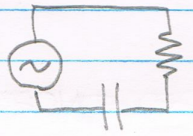
ω frequency, this is necessary for record players and is slightly different in NA & Europe

R-CIRCUIT



FIRST CONSIDER A RESISTOR IN AC-POWER. NOW, INSTEAD OF RESISTING BY $I = \frac{V}{R}$ WE GET $I = \frac{\mathcal{E}_{max} \sin(\omega t)}{R}$

RC-CIRCUIT



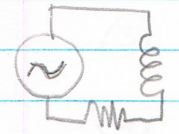
NORMALLY WE WOULD USE THE EQUATION $I = \frac{\mathcal{E}}{R}(e^{-t/RC})$, NOW, HOWEVER THE CURRENT =

$$I = \frac{\mathcal{E}_{max} \cos(\omega t)}{X_c}$$

X_c IS CALLED CAPACITIVE REACTANCE AND IS EQUAL TO $\frac{1}{\omega C}$ WHICH IS IN OHMS.

THE CURRENT HERE 'LEADS' THE EMF.

RL-CIRCUIT

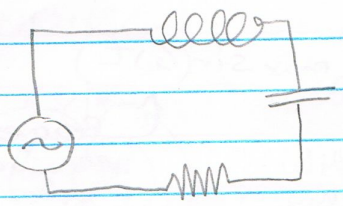


NORMALLY, WE WOULD USE THE EQUATION $I = \frac{\mathcal{E}}{R}(1 - e^{-tR/L})$, NOW, HOWEVER THE CURRENT=

$$I = -\frac{\mathcal{E}_{max} \cos \omega t}{\omega L}$$

WHERE $\omega L = X_L$ OR INDUCTIVE REACTANCE

FINALLY WE HAVE AN LCR-CIRCUIT THIS CIRCUIT LOOKS LIKE SO:



AN LCR, FREELY-OSCILLATING CIRCUIT IS AN SHO, WHEN DRIVEN BY A NEW FREQUENCY, OF AC POWER THERE IS THE POSSIBILITY FOR RESONANCE.

THE PHASE ANGLE IN AN LCR CIRCUIT IS $\frac{X_L - X_C}{R} = \tan\phi$ ∴ WE WANT THE AC-POWER TO HAVE THE SAME ϕ as ω IN ORDER TO GET RESONANCE.

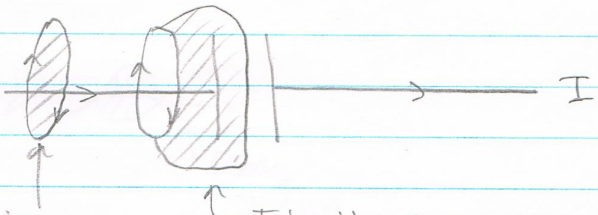
$$I_{max} = \frac{\mathcal{E}_{max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

THIS LOWER TERM IS CALLED IMPEDENCE

THE 'QUALITY' OF AN LCR CIRCUIT IS REPRESENTED BY: $\frac{1}{R}\sqrt{\frac{L}{C}}$ ETHIS IS HIGH RESONANCE IS DELICATE.

UNDERSTANDING THE DISPLACEMENT CURRENT

THE PROBLEM IS THAT AMPERE'S LAW APPEARS TO BREAK DOWN WITH CAPACITORS



if I examine this surface, magnetic force line integral is equal to current passing through HOWEVER!

Take the same cross section and slice between the capacitor. No 'current' yet there is clearly a magnetic field.

THE SOLUTION TO THIS PROBLEM IS A VIRTUAL DISPLACEMENT CURRENT FLOWING BETWEEN THE PLATES.

$$I_{displacement} = \underbrace{\varepsilon_0 \frac{d\Phi}{dt}}$$

THE DISPLACEMENT CURRENT IS NOT THE CHANGE/MOVEMENT OF CHARGED PARTICLES, BUT THE MOVEMENT OF ELECTRIC FLUX.

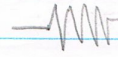
THIS GIVES ALL 4 MAXWELL EQUATIONS:

1. $\oint E \cdot dS = \frac{Q}{\varepsilon_0}$ ← GAUSS, ALL FLUX IN A CLOSED SURFACE = # OF CHARGES
2. $\oint B \cdot dS = 0$ ← NO MAGNETIC MONOPOLES (WHAT GOES IN, MUST COME OUT)
3. $\oint E \cdot dl = -\frac{d\Phi_B}{dt}$ ← A CHANGE IN MAGNETIC FLUX PRODUCES A CURRENT IN THE BOUNDARY
4. $\oint B \cdot dl = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi}{dt}$ ← THE CURRENT PLUS 'FLUX' CURRENT IS THE MAGNETIC FORCE AROUND A LOOP ON THE BOUNDARY.

Hilroy

UNDERSTANDING CIRCUIT ELEMENTS

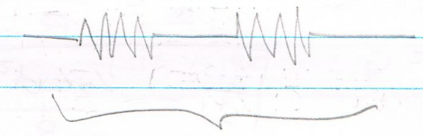
THE FOLLOWING INCLUDE POSSIBLE CIRCUIT ELEMENTS WITHIN A CIRCUIT DIAGRAM:



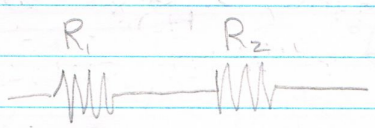
RESISTORS

RESISTORS DISSAPATE POWER IN A CIRCUIT

$$V = IR$$



2 RESISTORS IN PARALLEL DISSAPATE POWER THE SAME AS DOUBLE THE RESISTANCE OF THE CIRCUIT



THE POWER DISSAPATED IN EACH RESISTOR

$$P_n = I^2 R_n$$

CURRENT IS ITSELF A FUNCTION OF EMF AND THE NUMBER OF RESISTORS, CAPACITORS, SELF-INUCERS OR AC-CURRENT SOURCES

THE WAY TO FIGURE OUT THE POWER IS TO FOCUS ON TWO THINGS,

(1) HOW MUCH CURRENT IS FLOWING

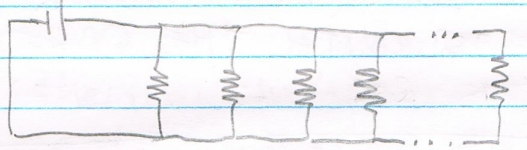
-AND-

(2) WHAT IS THE RESISTANCE

BECAUSE $I = \frac{V}{R}$ AND $P = I^2R$

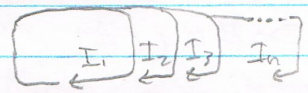
$P = \frac{V^2}{R}$ ← THEREFORE, MORE POWERFUL SOURCES ARE IN LOWER RESISTANCE

A 100 W BULB HAS LESS RESISTANCE THAN A 60 W BULB.



CONSIDER N RESISTORS IN PARALLEL. WHAT IS THE CURRENT? WHAT IS THE POWER CONSUMPTION?

THINK FIRST THAT THE CURRENT SPLITS N TIMES



THEREFORE THE TOTAL CURRENT IS THE ADDITION OF EACH PARALEL CIRCUIT LENGTH

$$I = I_1 + I_2 + I_3 + \dots I_n$$

IMAGINE A PIECE OF WOOL SPLITTING INTO EQUAL THREADS FOR EACH SUB-CURRENT

BY THIS REASONING EACH SUB-CURRENT IS EQUAL TO $I_i = \frac{V}{R_i}$ ← THE POTENTIAL DIFFERENCE IS THE SAME ← THE RESISTANCE IS UNCHANGED

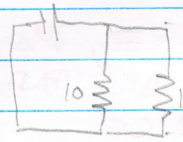
$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \dots \frac{V}{R_i}$$

THIS MEANS THE TOTAL RESISTANCE R_t IS EQUAL TO

$$R = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots \frac{1}{R_n}\right)^{-1}$$

Hilroy

HOW CAN WE MAKE SENSE OF THIS RESULT?



NET RESISTANCE $= \left(\frac{1}{10}+\frac{1}{10}\right)^{-1}$

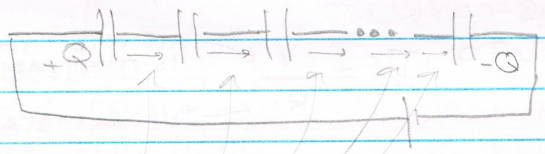
$= 5\Omega$

THE RESISTANCE IS LOWER, SO THE CURRENT FLOWS FASTER, HOWEVER IT BRANCHES AT EACH RESISTOR WHERE IT ONLY RESISTS WITH THE PARTIAL CURRENT.

UNDERSTANDING CAPACITORS IN SERIES AND PARALLEL

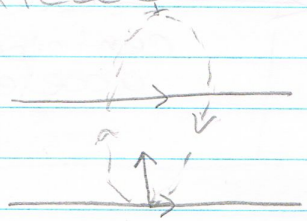
WHEN PUT IN PARALLEL, THE CAPACITOR INCREASES AS EACH ACTS LIKE AN EXTRA CONTAINER TO PUT CHARGE IN.

$$\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots \frac{1}{C_n} \leftarrow \text{why?}$$



each 'shuffles' the charge to match N times, lowering the capacity on each.

UNDERSTANDING MAGNETISM ON CURRENTS AND PARTICLES



↖ this wire creates a B field like so

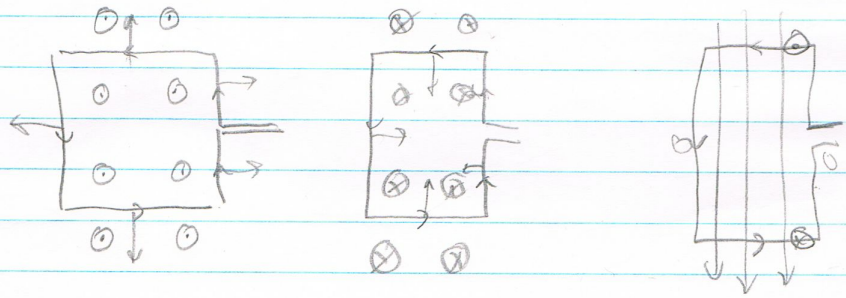
~~The~~ The effect of the B-field is

$B \times v$

The way to determine the ~~effect of a~~ magnetic field is to take your right hand and put your thumb in the direction of the current, the curl of your fingers are the B field

The way to determine the effect of a magnetic field on a current is to point with your index finger in the B-field, thumb in the current, ∴ other three will be direction of force.

Torque on a current loop



$\tau = \mu \times B$

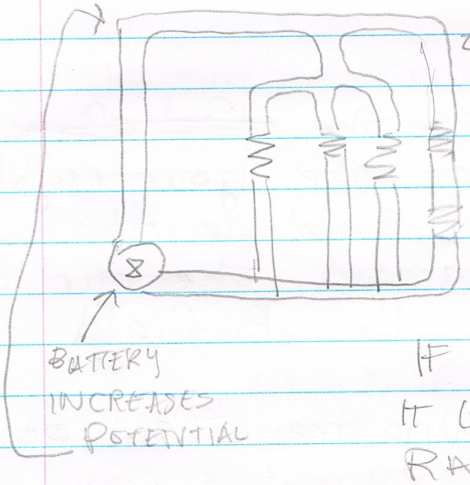
↑

Txlw

Hilroy

$$a = \frac{v^2}{R}$$ ← Formula for centripetal acceleration

Volts = Potential Change

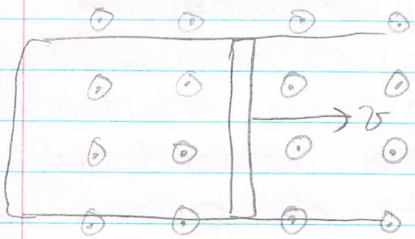


← THEN, UNLESS THERE IS ANOTHER BATTERY, IT MUST "FALL" THROUGH EACA CIRCUIT PATH RETURNING TO THE BASE

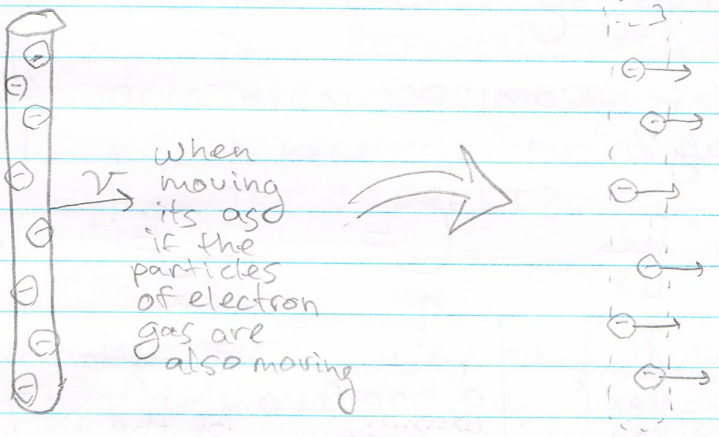
BATTERY INCREASES POTENTIAL

IF IT WASN'T ALL USED UP, IT WOULD BE A CONTINUOUSLY RATCHETING PUMP. IF IT

UNDERSTANDING INDUCED EMF



Imagine a conducting rod is moving through a uniform magnetic field and is connected to two wires on its ends

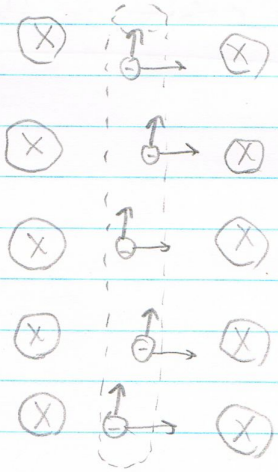


Now we know that a moving charged particle experiences a force in a B-field equal to

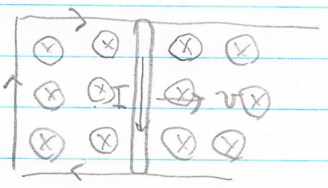
$$F = qv \times B$$

Therefore each electron receives a force of

$$-ev \times B$$



As the electrons are accelerated upward this creates a current in the downward direction



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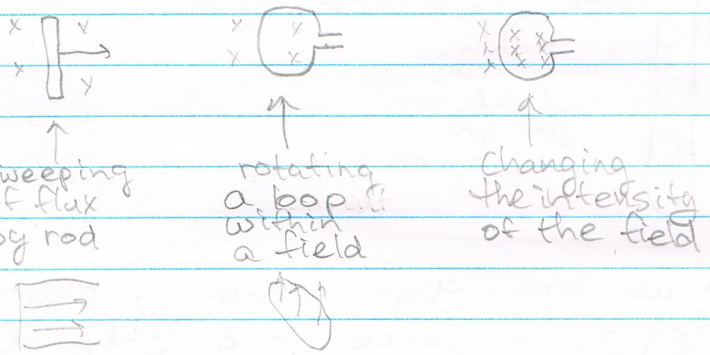
This potentialdifference created is called motional EMF, because the rod is moving.

The intensity of the induced emf is equal to the rate of sweeping of magnetic flux.

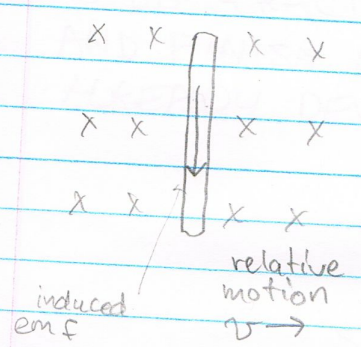
This can occur in more than one way:



In a betatron the field is increased in such a way to accelerate an electron, or charged particle. However, as it increases, the acceleration increases in step so the radii of orbit is the same.

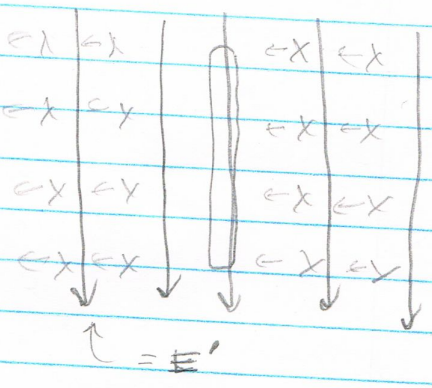
UNDERSTANDING INDUCED ELECTRIC FIELD

WE UNDERSTAND WHY A ROD MOVING THROUGH A MAGNETIC FIELD GENERATES A CURRENT. NOW, HOWEVER CONSIDER THE SAME ROD FROM A STATIONARY REFERENCE FRAME:



IF THE ROD ISN'T MOVING FROM THIS REFERENCE FRAME (AND AS SUCH THERE CANNOT BE AN IMPACT FROM A MAGNETIC FIELD) WHAT CAUSES THE CURRENT?

WELL IMAGINE THE MAGNETIC FIELD MOVING, IN ORDER TO SATISFY THE RELATIVITY, IT MUST INDUCE AN ELECTRIC FIELD



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